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Abstract

A product has a social (network) dimension when its use involves interaction between people. We analyse a monopoly market for social goods. When a good enabling more efficient interaction is introduced in the market, consumers must coordinate their decisions over switching to the more efficient medium or sticking with the legacy system. This coordination problem has multiple equilibria preventing determinate equilibrium analysis. We do a comprehensive analysis on the conditions for equilibrium uniqueness. Multiplicity is removed by high consumer heterogeneity under perfect information. Consumer heterogeneity must be real, in the sense of sufficient bandwidth of the consumer distribution. With incomplete information we can use global games techniques to eliminate multiplicity. Real heterogeneity between consumers can be minimal, but uniqueness presupposes a possibility that some people have very high or low valuations. Thanks to uniqueness, equilibrium analysis gives determinate predictions. The monopoly price is higher under incomplete information. Incomplete information regime yields more straightforward comparative statistics on profits and consumer surplus. Profits are decreasing in uncertainty. Consumer surplus increases in uncertainty, only if the level of uncertainty is already high.

JEL Classification: Coordination, information, heterogeneity, global games, networks, monopoly.

Keywords: D42, D82, L14.

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1 Introduction

It is a well-known fact that network externalities cause multiple equilibria in economic models. Indeterminancy arising from multiplicity of equilibria has been incorporated in the theory in the forms of de facto standards and bandwagon strategy profiles. The line of these models can be traced back to Arthur (1989), David (1985), Farrell & Saloner (1985, 1986), who study technology adoption, and Katz & Shapiro (1985) who look at brand competition and compatibility between networks. Those seminal papers and subsequent literature accepting indeterminacy as a characteristic of economic networks, suggests that market structures in network industries are determined by random exogenous events. In this paper, we argue that this is partial truth. Uniqueness of equilibrium follows endogenously in a broad range of heterogenous network models, eliminating the role of random events in equilibrium selection. Drawing from the theoretical work on coordination games, we unify the rather ad hoc solutions to the multiplicity problem employed in the network literature by showing how (i) under perfect information, the achievable intrinsic utility, which is independent of the network size, must be the dominant criterion in the consumer's buying decision, (ii) under incomplete information, uniqueness is independent of the (relative) strength of externalities, but requires so called dominance regions of strictly dominant strategies.

We analyse a monopoly pricing problem of a network product. The utility from the product increases as the number of consumers who buy increases. A version of the model with homogenous consumers produces multiple equilibria with rational (thus fulfilled) expectations[1] . Multiplicity is a particularly troublesome problem in this setting, as it hampers equilibrium analysis. The firm's pricing strategy is equilibrium-specific; hence, in order to derive the optimal price, the analysis must focus on one equilibrium at a time. In addition, there is the extra burden of argumentation in favour of a particular equilibrium. Since, the arguments are inevitably exogenous to the model, we do not learn much on firm behaviour at the end. If the exogenous factors that select the

[1] Rational expectations require that the expected network size matches the true realised network size. Some dynamic games adopt myopic expectations. Myopic consumers expect the network size to remain at the current size. Myopism eliminates multiplicity, but at the cost of (too) severe limitation on consumer behaviour.

equilibrium are known, why should not the firm take the effort to influence them rather than surrender to the situation - inshallah?

Work on network externalities has tried to overcome the problem of multiplicity often by simply analysing only the most interesting equilibrium, by resorting to arguments that are exogenous to the model, or by restricting the strength of externalities. Exogenous arguments that are used to eliminate multiplicity seldom are more than illustrations of expectations and equilibria. Farrell & Katz (1998) provide a compelling discussion about different behavioural cues which align consumers' expectations in favour of a particular (fulfilled expectations) equilibrium. They show how a particular equilibrium is selected when consumers' expectations track surplus, quality, or when expectations favour stubbornly one firm. Still, the problem of multiplicity is not solved truly satisfactorily as the motivation for their selection process is exogenous. Uniqueness of equilibrium that can be reached through exogenous argumentation is, in fact, based on an assumption.

Earlier literature on pricing of network goods has treated the problem of multiple equilibria in a case-by-case manner. Baake & Boom (2001) analyse a quality differentiated duopoly with heterogenous consumers with respect to intrinsic utility. They identify that the Nash equilibrium is unique only if consumers evaluate the competing products chiefly in terms of quality (opposed to perceived network sizes). However, the condition for quality difference they provide is better interpreted as a requirement for a sufficient level of consumer heterogeneity, because quality, in their model, affects equally intrinsic utility and network size dependent utility. Cabral et al. (1999) get a unique interior equilibrium by assuming that the discount factor and the parameter measuring network externalities are not "too large". Bental & Spiegel (1995) analyse only the non-zero equilibrium associated with (non-zero) fulfilled expectations. De Palma & Leruth (1996) obtain a unique equilibrium in a duopoly model, where consumers value network externalities differently, by allowing the firms to commit to production levels. Economides (1996) studies an oligopoly market with network externalities. In line with general results, uniqueness of equilibrium in his model also hinges on the magnitude of network externalities. Equilibrium is unique (and interior) only if the externalities function is concave with the marginal externality sufficiently small.

In other cases, where network effects, *per se*, are not the primary subject of analysis, multiplicity is abstracted away by assuming covered markets and high level of product differentiation for example. This route has been successfully used in the analyses of (physical) network-based industries such as telecommunications networks (see Armstrong 1998, and Laffont et al. 1998a and 1998b).

A model with demand-side network externalities is essentially a coordination game where agents' actions are strategic complements à la Bulow et al. (1985). A coordination game with perfect information and homogenous players has multiple equilibria. In most circumstances, equilibrium uniqueness can be reached if either homogeneity or perfect information is removed.

A coordination game with increasing returns to scale and perfect information can have a unique equilibrium if players are sufficiently heterogenous (Herrendorf et al. 2000). The trade-off of high heterogeneity is that we must impose quite a stringent set of conditions on the magnitude of network effects. An alternative route to uniqueness is to limit agents' capacity to observe information. Recent work on global games has developed a theory that yields equilibrium uniqueness endogenously in coordination games[2] . Global games originate in Carlsson & van Damme (1993); the technique is surveyed and advanced in Morris & Shin (2003). In global games, agents obtain correlated imperfect signals of the true state of some underlying economic fundamental. Given the signals, agents establish beliefs about the state of the fundamental and, more importantly, about other agents' beliefs of the fundamental and even higher order beliefs. Global games provide an elegant way to achieve uniqueness in situations where homogeneity between agents and common knowledge result in multiple equilibria.

Whether information is perfect or incomplete, the key to uniqueness is the same. Uniqueness

[2] Mason & Valentinyi (2003) derive conditions for existence of unique equilibrium in a larger set of incomplete information games that includes global games. They show that unique equilibrium exists if the (conditional) heterogeneity and correlation between consumer types are sufficiently high. Their result does not depend on strategic complementarities or dominance regions that are essential in global games.

follows when one group of people play one action as a strictly dominant strategy at the same time as another group of people play a different action also as a strictly dominant strategy. The surviving equilibrium is a switching strategy with a uniquely determined cut-off point. The advantage of global games is that necessary conditions on agent heterogeneity are less restricting. We can allow high (relative) network externalities; with perfect information externalities must be bounded. Under perfect information, heterogeneity must be "real" in the sense that the distribution of consumers must have sufficiently broad support. In global games, uniqueness is guaranteed with sufficiently dispersed prior distribution of the underlying fundamental. In other words, uniqueness requires only a *possibility* that some consumers obtain extremely low or high signals. Real heterogeneity can be relatively small. In fact, many global games applications study the case where signals become perfectly accurate at the limit so that heterogeneity between agents diminishes to zero.

In this paper, we analyse monopoly pricing under demand-side network externalities. We model a market where consumers have a need to interact with each other. The monopoly launches a new device that constitutes an efficient medium for interaction. The fax machine, mobile phone, or on-line game console are examples of the product we have in mind. The consumer is able to interact with the new device only with those people who have bought the device as well. Hence, there is a coordination problem between consumers: whether to switch to the new medium or to stick with the legacy system. Effectively, the consumer benefits the more people buy the new product. We remedy the inherent multiple equilibria problem in two alternative ways. First, we solve the problem under perfect information. We show how network effects must be limited if uniqueness is to be reached. Second, we limit consumers' capacity to observe information. Uniqueness is derived using global games techniques.

We analyse the differences between the two ways to reach uniqueness. Perfect information regime turns out analytically more complicated. We need to keep track about various possible states of the world. More seriously, unique equilibrium is reached only if network usage does not drive consumers' decision making. For a model of network effects this constraint is troublesome. Global games are analytically more applicable to our model. We also argue that the case of incomplete information better characterises the real world. When a new device is launched, people are not able to tell how much utility the device yields to other people. This informational asymmetry is aggravated the more drastic innovation the new device is.

The benefit of uniqueness is that analysis on firm behaviour becomes clear-cut. We derive the optimal two-part tariff structure for the monopoly, and analyse the effects of a marginal change in heterogeneity (i.e. uncertainty under incomplete information) on the equilibrium. The optimal unit price is increasing in consumer heterogeneity under perfect information. If information is incomplete, the price is independent of uncertainty (heterogeneity). When we compare prices across the informational regimes, we see that the optimal unit price tends to be higher under incomplete information.

The effect of a marginal change in heterogeneity is ambiguous on profits and consumer surplus under perfect information. The results are more clear-cut under incomplete information. The firm's expected profits increase as uncertainty is reduced. The effect on expected consumer surplus depends on the absolute value of uncertainty. The effect of a marginal change in uncertainty is positive if the change is aligned with the absolute value That is, if uncertainty is high, then further uncertainty is of good. Similarly, the expected consumer surplus increases when there is little uncertainty and we further reduce uncertainty.

The global games approach has been successfully used in a number of macroeconomic and financial problems. Morris & Shin (1998) analyse a model of speculative currency attacks. Heinemann et al. (2004) test experimentally this kind of a currency attack model. Their results support the switching strategy equilibrium predicted by global games. They also report results that accord with the comparative statics of global games. Englmaier & Reisinger (2003) apply global games to an economic development framework. Morris & Shin (2004) and Rochet & Vives (2004) study solvent but illiquid financial institutions. Morris & Shin (2004) focus on the question, how investors' beliefs affect the price of debt; whereas Rochet & Vives (2004) explain how the central bank can prevent bank runs with lender of last resort facilities. Myatt & Wallace (2002) analyse public goods provision with global games techniques. They use open source software as an example. Chwe (1998) provides empirical observations that support global games' predictions. He finds that goods with social (network) externalities advertise "more on more expensive popular [TV] shows because viewers of popular shows know that many other people are also watching (Chwe 1998)".

The present paper, together with Farhi & Hagiu (2004) and Argenziano (2004), are the first applications of global games in network economics. These models are also first to endogenise the payoffs with a pricing problem. Both Farhi & Hagiu (2004) and Argenziano (2004) study a platform competition with pure membership externalities. Our model differs from those models in that we study a market with membership and usage decisions. We analyse a monopolist that sets two-part tariffs whereas Farhi & Hagiu (2004) and Argenziano (2004) analyse a Bertrand duopoly with linear prices.

Our model is also related to the product differentiation literature. We propose that the case where the noisy signal of the underlying state directly enters consumer's utility function, is a kind of horizontal differentiation outlined by Hotelling (1929). Telecommunications network competition models by Armstrong (1998) and Laffont et al. (1998a and 1998b) are also related to our model. We analyse similar demand schemes of network usage as these models do, but we extend the model to analyse also the question whether to subscribe to the service or not.

Finally, the literature on capacity constraints is also relevant. In our two-period model, buying decisions are done in the first period, and subsequent usage decisions in the second period. Usage may be constrained by the number of consumers who bought the product in the first period. Hence, the monopolist creates "capacity" for itself.

2 Model

We begin with an informal description of the model. The market consists of consumers and a monopoly firm (an innovator). The firm sells a novel device that constitutes an efficient medium for interaction. Interaction usage is the source of a coordination problem between consumers: the consumer needs to evaluate the proportion of other people who acquire the device. The problem we solve is how the firm sets a price for a new network product. The efficiency of the device is an objective measure. Everybody agrees that all pre-innovation interaction can be mediated by the new device, and the quality of interaction is improved. Capacity to mediate interaction is the main functionality of the product, but it also provides standalone services that are used independently of other consumers. The utility from the product is thus split into usage and intrinsic utilities. Usage utility is generated by efficiently mediated interaction between people, and it presents positive network externalities as it increases with the number of people who buy. Standalone services yield intrinsic utility. Intrinsic utility may include also a status-enhancing type of utility, any utility derived from use with older generation services (backward compatibility), and any non-direct benefits of being a member of the network (including higher-order interaction benefits[3]).

Consumers are horizontally differentiated according to their perception of the intrinsic value. Differentiation captures the idea of consumer satisfaction with product's technical performance and status-related aspects. Lower consumer types find technical performance rather poor. High types are those who like how the machine works (plus probably get high satisfaction from ownership). Intrinsic utility does not have to be positive in relation to older generation products. In fact, depending on the informational regime, equilibrium uniqueness requires a possibility that some consumers get negative intrinsic utility. We elaborate this point at the end of this section.

Why is intrinsic utility subjected to differentiation while usage is not? On the one hand,

[3] Higher order interaction benefits comprise utility from interaction taking place between one's friends' friends, between friends' friends' friends, and so forth.

usage utility is directly associated with the people who interact, or more precisely, with the social relation the interacting parties have. The device is a mere medium, which does not influence the value of the social relation. We assume that each consumer has equally valuable social relations and the improvement in efficiency is identical for all. On the other hand, how different consumers get utility from the novel features of the device are captured in the intrinsic utility. Clearly, the capacity to use and the attitude towards new technology differ between people.

The new device can be used in interaction only if both parties have bought the product. For example, let consumer i have a need to interact with j. If both i and j have bought the device, then they can use it. If either i or j does not have the product, then they use conventional ways to interact. Interaction is not anonymous. From consumer i's point of view, interaction with j is a different good from interaction with k. Consequently, inability to interact with j cannot be compensated by interaction with k. This is what we call an *exogenous social network structure*. Each social relation is perceived as independent from other relations and the relations have different values. The following clarifies the exogeneity assumption. Let the new device be a game console with a play-over-the-Internet capacity. Consumer i has a friend j and a distant (and boring) relative k. She wants to play games online with j, but not with k. It happens that i and k have bought the device, but j has not as he likes to play traditional board games face-to-face. Inability to use the game console with j, does not increase i's desire to play with k.

In the main analysis, we assume that each consumer is interested in interacting with the whole population. In the supplementary section, we analyse a case where each consumer is interested in interaction with only a sub-set of total population, called neighbours in the social relations literature. We show in the supplement that the "global" and "local" interaction models coincide when all consumers have the same number of neighbours.

The reader may want to keep in mind the following two real world examples. First example is online gaming. Sony PlayStation 2, Microsoft Xbox and Nokia N-Gage consoles all have standalone and interaction usage features. Players can play alone against the console's computer or against other players on the same console. In addition, console manufacturers run platforms that allow people to play over the Internet against other people (Sony's Central Station, and Xbox Live; N-Gage allows to play over the air directly). The requirements are that the players buy the console, and that they have a broadband connection to the Internet. Moreover, both Xbox Live and Sony Central Station enable players to talk with each other during a game session. So, the console in fact enables sessions that are very active social events at best. Firms also offer additional services and content on the platforms. One can think that the underlying social network consists of players who play the same games or play repeatedly together. It is also usual that people swap/borrow/trade games with their friends. A person who thinks about buying a game console, takes into account how many games that particular console has in supply and what is the quality of the console. Another point he bears in mind is whether his friends have the same console brand so that he can play with/against them.

The second real world example is mobile telecommunications. Mobile phones can be also used nowadays for checking latest news and e-mails, or to listen to the radio and music, and even to watch television. These features create intrinsic value for a phone. The main value driver, of course, is the possibility to talk with friends and send them messages independent of time and place.

Let us return to the question how the intrinsic utility can be negative with respect to older generation products, while the interaction utility is valued objectively positive. An example clarifies this point. "Mobility" is the principal improvement of mobile telecommunications with respect to fixed line telephony. Being able to call and to be called independent of time and place is an objectively measured improvement (you can always keep the phone switched off whenever you wish!). However, mobile phones tend to be small in size and their use can therefore be very difficult for e.g. old people. The size factor is positive for most people, but it can also be negative. Alternatively, some people believe that mobile phones emit radiation harmful to the brain. Other people fear that third parties (big brother) can secretly monitor the user. Whether it is due to the fear of

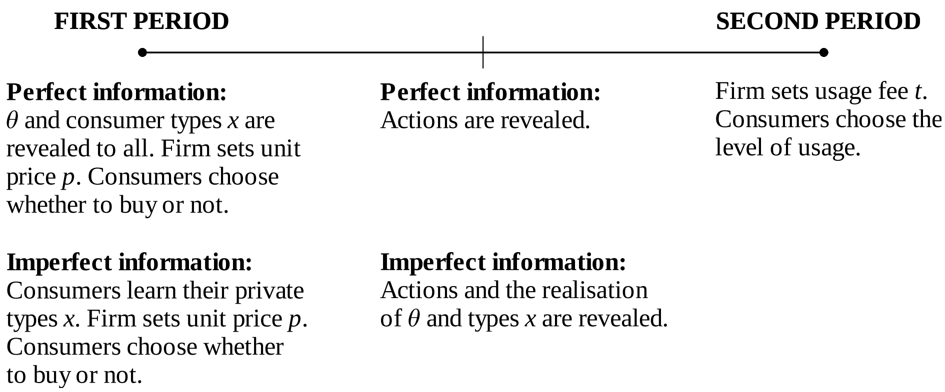

Figure 1: Time Line.

brain tumors or malicious surveillance, some people may be reluctant to carry a mobile phone, even if they get one for free. Obviously, mobile phones have been fairly successful, and a negative intrinsic value can apply to a handful of people at most; but for uniqueness of equilibrium that is enough.

2.1 Players and timing

There are I consumers in the market. We normalise $I = 1$ and treat it as continuous. The fundamental intrinsic value of the product θ is drawn from a uniform distribution $F(\theta)$. Consumer types x are distributed around the fundamental according to a uniform distribution $G(x \mid \theta)$.

Timing is summarised in figure (1). The game has two periods. In the first period, the firm sets unit price p, and in the second period it sets usage fee t. Fixed costs for the firm are assumed zero. There is a trade-off in choosing the optimal price. A low unit price facilitates coordination between consumers and increases expected second period profits, but it erodes first period margins.

The first period problem for consumer $i \in I$ is to choose action $a_i \in \{B, N\}$, where $B =$ buy the device and $N =$ do not buy. If the consumer chose $a_i = B$ in the first period, then he needs to decide how much he uses it in the second period. Those consumers, who did not buy, collect the reservation utility of zero and make no further decisions. Usage is possible only among consumers who have bought the product. Interaction needs two people, but we assume that only the person paying for the usage gets utility. "Reception" is cost-less and yields zero utility, or any possible positive utility is included in the intrinsic utility. Since usage is a binary operation, it is likely that in a given social relation, both consumers pay for usage and get utility.

The second period is a standard deterministic utility maximisation problem for the consumers, and a deterministic profits maximisation problem for the firm respectively. In the first period, where the coordination problem is in effect, the model is exposed to two informational regimes. First, if information is common to all players (consumers and the firm), then all uncertainty is resolved already at the beginning of the game. The game is deterministic throughout. Second, if information is incomplete in the first period, consumers observe noisy signals of θ which correspond to consumer types. The realisations of the signals are private information, but the distributions $F(\theta)$ and $G(x \mid \theta)$ are common knowledge. The firm observes nothing and resorts to the prior $F(\theta)$. Such informational asymmetry is due to our interest in those cases where consumers know their needs better than the firm. All uncertainty is resolved before players make their second period decisions.

2.2 Second period

Social relations are unequal in terms of interaction utility. Consumers arrange (mentally) the whole population in descending order in terms of desire for interaction. The person who is the most desirable interaction partner gets index 0, and the least desirable person gets index 1. Then, consumers decide which fraction of population they want to interact with for a given price. The underlying social network is exogenous so that interaction needs are independent of who buys the device or of the counter part's utility and respective ranking of interaction partners. Consumer i may rank j high in terms of desire to interact, but j can rank i independently high or low. A missed chance of usage because the counter part has not bought the product, is not compensated by increased usage among other social contacts. As a result, the ordering of desired usage for each consumer is exogenous, which guarantees that consumers are symmetric with respect to usage demand in the second period.

Let $\alpha_i \in [0,1]$ be the marginal person consumer i wants to interact with (i.e. the last person worth interaction for a given usage price). $\alpha_i = 1$ means that i wants to interact with the whole population. Symmetry of social contacts guarantees $\alpha_i = \alpha$ for all $i \in I$. This formulation makes it possible that the consumer would like to interact with more people than who have bought the product. Hence, if the actualised first period demand is low, the consumer may be constrained into a sub-optimal level of usage. We sacrifice some generality and assume that the marginal utility is linear, but it will become evident that any function with decreasing marginal utility yields qualitatively identical results. If fraction $q \in [0,1]$ has played $a = B$ in the first period, then by the law of large numbers, q is also the probability that a particular person has bought the product. Due to exogenous social network, the marginal utility from interaction with the social contact indexed α is $\frac{\partial \lambda(\alpha,t)}{\partial \alpha} = q\left(1 - \alpha - t\right)$, where t is the (scaled) price for usage. Integration gives the expected usage utility

$$\lambda\left(\alpha, t, q\right) = q\left(\alpha - \frac{1}{2}\alpha^2 - \alpha t\right),$$

with the integration constant equal to zero. Because only the proportion q of the population has bought the product, the consumer cannot use the device with more than q people. Hence, the consumer's second period objective is

$$\max_{\alpha}\left\{\lambda\left(\alpha, t, q\right)\right\}, \text{ s.t. } \alpha \in [0, q].$$

The optimal level of usage is

$$\alpha^*\left(t, q\right) = \min\left\{1 - t, q\right\}. \tag{1}$$

Consumers wish (but cannot if $q < 1$) to interact with the whole population when the usage price is zero. When the usage price tends to one, consumers choose not to use the device in interaction (and enjoy only of the intrinsic utility).

The firm's second period problem is to maximise usage profits by setting the usage fee t.

Demand is given by equation (1), which has two kinks: one at $t = 1 - q$, and the other at $t = 1$. At the latter point demand hits zero.

The firm's total 2-period expected profits are $\mathbb{E}(\Pi) = \mathbb{E}(\Pi_1) + \mathbb{E}(\Pi_2)$, assuming no discounting. In the second period, first period profits and the proportion of consumers who bought the product are fixed, and all uncertainty has been resolved. So, the second period profits are

$$\Pi_2 = q\alpha^*(t, q)(t - c_a),$$

where $c_a \in [0, 1]$ is the unit cost of service.

The optimal fee is always positive. More importantly, the firm charges always a usage fee such that the consumers are maintained at an efficient usage level, so that an increase (decrease) in price causes a decrease (increase) in demand. To see this, assume that t is such that consumers are constrained in their usage, i.e. $1 - t > q \Leftrightarrow t < 1 - q$. Then, the firm could increase its price t up till point $t = 1 - q$ without triggering a decrease in demand. A similar argument holds for the situation where the firm charges a price $t \geq 1$ so that demand is zero. In this case, it would pay off to reduce the price below one $t < 1$. These observations allow us to write the firm's second period problem as

$$\max_t \{q\alpha^*(t, q)(t - c_a)\}, \text{ s.t. } t \in [1 - q, 1[.$$

The optimal usage fee is

$$t^* = \max\left\{\frac{1}{2}(1 + c_a), 1 - q\right\}, \tag{2}$$

with the interior solution $t^* = \frac{1}{2}(1 + c_a)$ satisfying second order conditions, $\frac{\partial^2 \Pi_2}{\partial t^2} = -2q < 0$. When the optimal usage fee (2) is plugged back into the second period profits, we get the value functions

$$\Pi_2^*(c_a, q) = \begin{cases} q\pi_2^{**}(c_a), & q \geq \frac{1}{2}(1 - c_a) \\ q\pi_2^*(c_a, q), & q < \frac{1}{2}(1 - c_a) \end{cases}, \tag{3}$$

where $\pi_2^{**}(c_a) = \frac{1}{4}(1 - c_a)^2$ and $\pi_2^*(c_a, q) = q(1 - c_a - q)$. Double star indicates that the monopolist is at the interior (unconstrained) solution and single star that the monopolist is at the corner solution. Naturally, we have $\pi_2^{**}(c_a) \geq \pi_2^*(c_a, q)$.

Because the firm keeps consumers at the efficient level of usage, $\alpha^*(t^*, q) = 1 - t^*$ and $\lambda^*(\alpha^*(t^*, q), t^*, q) = \frac{1}{2}q(1-t^*)^2$ hold when t is optimally chosen. Substituting t^* in the expected indirect usage utility, we get

$$\lambda^*(q) = \begin{cases} \frac{1}{8}q(1-c_a)^2, \text{ if } q \geq \frac{1}{2}(1-c_a) \\ \frac{1}{2}q^3, \text{ if } q < \frac{1}{2}(1-c_a) \end{cases}. \tag{4}$$

This concludes the analysis of the second period. Next we study the first period when players have perfect information. In the section following, we analyse the incomplete information case.

3 Analysis with perfect information

The consumer's payoff increases in the number of other people buying, exhibiting positive network externalities. With relatively homogeneous consumers and perfect information, the model produces multiple equilibria, typical to network externalities models. However, as Herrendorf et al. (2000) illustrate, heterogeneity between consumers can yield uniqueness in games which homogenous versions produce multiple equilibria. In our model, uniqueness requires sufficient amount of heterogeneity between consumers with respect to intrinsic utility. In other words, uniqueness results only when network externalities are relatively low. Endogenous pricing is not a remedy to the multiplicity problem. Effectively, when network externalities are relatively low the monopolist sets a price which guarantees a unique equilibrium, but for high network externalities pricing involves multiple equilibria.

The fundamental intrinsic utility θ is drawn from the uniform distribution $F(\theta)$ over the support $[-M, M]$. When θ is the realisation, consumers obtain i.i.d. private values x according to the conditional uniform distribution $G(x \mid \theta)$ over $[\theta - \epsilon, \theta + \epsilon]$. Types x and the value of the fundamental θ are perfectly observed by the consumers and the firm at the beginning of the first period.

The payoffs for different actions, when the consumer is of type x and the proportion q of population play $a = B$ is provided in the table below

	others play B	others play N
$a = B$	$x + \widehat{\lambda}^* (q) - p$	$x - p$
$a = N$	0	0

The unit price for the product is p. We have used $\widehat{\lambda}^* (q) = \frac{1}{q} \lambda^* (q)$ for the indirect usage utility. When the fraction q of population buy the product, consumer's expected utility for $a = B$ when he is of type x is $u(x, q, B) = x + \lambda^* (q) - p$. If $x - p > 0$, then $a = B$ is strictly dominating strategy. Action N is strictly dominating strategy when $x + \widehat{\lambda}^* (1) - p < 0$. Action B is the best response, if the fraction of other people playing B is at least $q \geq \frac{p-x}{\widehat{\lambda}^*(q)}$. Because the reservation utility from $a = N$ is zero, the payoff gain from action $a = B$ versus $a = N$ is[4]

$$v(x, q, p) = x + \lambda^* (q) - p. \tag{5}$$

Denote by Γ the coordination game of perfect information with I consumers, pure actions $a \in \{B, N\}$, and payoff $v(x, q, p)$. The payoff function (5) is continuous in its arguments, even at the cut-off point $q = \frac{1}{2}(1 - c_a)$ where the usage utility $\lambda^* (q)$ changes its shape. Function (5) is also differentiable, except at $q = \frac{1}{2}(1 - c_a)$. The payoff presents strictly increasing differences in x. Actions are strategic complements, because the payoff gain from choosing $a = B$ compared to $a = N$ is strictly higher when larger proportion of population choose $a = B$. Since the action set $a \in \{B, N\}$ is a compact subset of $\mathbb{R}$, the complementarity and continuity properties of $v(x, q, p)$ imply that Γ is supermodular (see e.g. Vives 2001 ch.2).

Supermodularity of Γ guarantees the existence of a Nash equilibrium (NE), which is solvable by iterated deletion of strictly dominated strategies. The equilibrium may not be unique, but it has a smallest and a largest element. Because actions are strategic complements the largest equilibrium element is Pareto dominating.

[4] The derived payoff function is essentially in line with the utility specification of Katz & Shapiro (1985), where consumers are differentiated in terms of intrinsic utility, and variable utility depending on the network size is the same for all buyers. De Palma & Leruth (1995) analyse the polar case where buyers have different valuations for the network benefits.

The payoff (5) actually depends on the consumer's expectations about other people's behaviour. In equilibrium, expectations are fulfilled, $\mathbb{E}_i(q) = q$ for all $i \in I$. When the consumer expects that proportion $\mathbb{E}(q) = q^e$ of people play B, he is indifferent between buying and not when his type is

$$\overline{x}(q^e, p) = p - \lambda^*(q^e). \tag{6}$$

The corresponding demand schedule is

$$q(p, q^e) = \begin{cases} 0, & \text{if } \overline{x}(q^e, p) > \theta + \epsilon \\ 1 - G(\overline{x}(q^e, p) \mid \theta), & \text{if } \theta - \epsilon \leq \overline{x}(q^e, p) \leq \theta + \epsilon \\ 1, & \text{if } \overline{x}(q^e, p) < \theta - \epsilon \end{cases} . \tag{7}$$

For a given pair (q^e, p), if there is a marginal type defined by (6), the type is unique because $v(x, q^e, p)$ is continuous and strictly increasing in x. More "optimistic" expectations reduce the marginal type, $\frac{\partial \overline{x}(q^e, p)}{\partial q^e} < 0$. This captures the correspondence between efficient coordination and the maximal NE.

Definition 1 *Nash equilibrium of* Γ *is the action profile*

$$\begin{cases} a^* = B \Leftrightarrow x \geq \overline{x}(q, p) \\ a^* = N \Leftrightarrow x < \overline{x}(q, p) \end{cases},$$

where expectations are fulfilled $\mathbb{E}_i(q) = q,$ *and* $\overline{x}(q, p) = p - \lambda^*(q)$ *for all* $i \in I$. *The maximal NE element corresponds to Pareto-efficient coordination.*

Because the NE action depends on other players' actions, the equilibrium is not necessarily unique. At the extremes, the firm could set prices for which no-one buys or everyone buys (granting negative prices). For intermediary prices, there are potentially multiple equilibria. The firm incorporates consumers' expectations in its price strategy and adjusts its price accordingly. In equilibrium everyone, including the firm, knows on which NE consumers coordinate, so the firm aligns its price with the particular NE of the coordination game that emerges.

Consumers who play the bandwagon strategy: "I buy only if you buy" are the cause of multiplicity. These consumers buy only if sufficiently many others buy. If coordination is efficient, then they in fact buy. In a coordination failure they do not buy; only those consumers who have a strictly dominating strategy to buy, will buy. Multiplicity of equilibria is ruled out when we allow sufficient level of heterogeneity between consumers[5] .

We allow negative unit prices, but we categorically rule out states θ that are prohibitively negative as well as prohibitively high unit production costs c_f in order to exclude those cases where the firm chooses to remain inactive. If the state θ is negative, it means that the firm may have to compensate some consumers by setting a negative price. Let $p^*(\theta, c_f)$ and $q^*(\theta, c_f)$ be the optimal price and quantity respectively for state θ and costs c_f. A prohibiting state-cost pair $\left(\theta^-, c_f^+\right)$ is defined implicitly by

$$\begin{aligned} 0 &\leq \pi_2^*\left(c_a, q^*\left(\theta^-, c_f^+\right)\right) < c_f^+ - p^*\left(\theta^-, c_f^+\right), \text{ if } q^*\left(\theta^-, c_f^+\right) < \frac{1}{2}(1-c_a) \\ 0 &\leq \pi_2^{**}(c_a) < c_f^+ - p^*\left(\theta^-, c_f^+\right), \text{ if } q^*\left(\theta^-, c_f^+\right) \geq \frac{1}{2}(1-c_a) \end{aligned} \tag{8}$$

For the state-cost pair $\left(\theta^-, c_f^+\right)$, first period losses outweigh second period profits.

Define price $\underline{p}$ as the solution to $v\left(\theta-\epsilon, q, \underline{p}\right) = 0 \ \forall q \in [0,1]$ where θ is the realisation of the fundamental. In words, the solution $\underline{p}$ is the lowest type's answer to question: "What is the lowest price that makes sure that even when everybody else buy, I still will not buy?" The answer is $\underline{p} = \theta - \epsilon + \frac{1}{8}(1-c_a)^2$ (to be precise, $\underline{p}$ leaves the lowest type indifferent). Now we have to distinguish between two cases: (i) (relatively) high network externalities and (ii) low network externalities. Network externalities are high if they dominate the intrinsic utility in the sense $v(\theta-\epsilon, q=1, p) > v(\theta+\epsilon, q=0, p) \Leftrightarrow \epsilon < \frac{1}{16}(1-c_a)^2$. When network externalities are high, price $\underline{p}$ exceeds the highest type's intrinsic valuation $\theta + \epsilon - \underline{p} < 0$.

Proposition 2 *Define*

(i) *Optimal monopoly price* $p^* = \arg\max\{\Pi(p)\}$, *where*

$$\Pi(p) = \begin{cases} q(p)(p-c_f) + q(p)\pi_2^{**}(c_a), & \textit{if } q(p^*) \geq \frac{1}{2}(1-c_a) \\ q(p)(p-c_f) + q(p)\pi_2^*(c_a, q(p)), & \textit{if } q(p^*) < \frac{1}{2}(1-c_a) \end{cases}.$$

[5] A perfectly homogenous version of the model at hand is where all consumers have intrinsic valuation $x = \theta$ (i.e. $\epsilon \to 0$). This homogenous version of Γ has a unique all buy equilibrium when price $p < \theta$. It has a unique no-one buys equilibrium for price $p > \theta + \frac{1}{8}(1-c_a)^2$. For intermediary prices $p \in \left[\theta, \theta + \frac{1}{8}(1-c_a)^2\right]$ there are multiple equilibria. The set of equilibria includes all buy and all not buy equilibria. In addition there is an equilibrium with fraction $q' \in]0,1[$ who play $a = B$ and fraction $1-q'$ who play $a = N$. The fraction q' depends on price p in the manner $\begin{cases} p < \theta + \frac{1}{16}(1-c_a)^3 \Longleftrightarrow q' = [2(p-\theta)]^{\frac{1}{3}} \\ p \geq \theta + \frac{1}{16}(1-c_a)^3 \Longleftrightarrow q' = \frac{p-\theta}{\frac{1}{8}(1-c_a)^2} \end{cases}$.

(ii) *Network externalities are high (low) relative to intrinsic utility when* $\epsilon \underset{(>)}{<} \frac{1}{16}(1-c_a)^2$.

With endogenous price setting:

1. *If network externalities are high, there are always multiple equilibria.*

2. *If network externalities are low, there is always a unique equilibrium.*

Proof. *In the appendix.* ■

We sketch the proof of Proposition 2 here. (1) Consider first the case where network externalities are high, $\epsilon < \frac{1}{16}(1-c_a)^2$. Assume first that coordination among consumers is efficient so that the maximal NE emerges (for a given p). Then the lowest price the firm will ever set is $\underline{p}$ defined above. This price corresponds to full demand. In general, monopoly reduces output, which implies that the optimal monopoly price can be above the price $\underline{p}$. Since the optimal price $p^* > \theta + \epsilon$ under efficient coordination, all consumers get negative intrinsic utility net of price. For a given price $p > \theta + \epsilon$, if coordination is efficient, the maximal NE emerges, but under total coordination failure, $q^e = 0$, no-one will buy. Both cases correspond to fulfilled expectations. In equilibrium, everybody knows which NE takes place. Hence, under the super pessimistic expectations $q^e = 0$, the firm adjusts its price downwards. The optimal price becomes $p^* < \theta + \epsilon$, which guarantees that the highest type has a strictly dominant strategy to buy. In sum, under high network externalities, the coordination game Γ has always multiple equilibria, and the optimal monopoly price is different for different NE. Efficient coordination supports the highest optimal price, and total coordination failure the lowest.

(2) Assume low network externalities $\epsilon > \frac{1}{16}(1-c_a)^2$. The lowest price the firm will ever set is $\underline{p} = \theta - \epsilon + \frac{1}{8}(1-c_a)^2$. Contrary to the case of high externalities, the lowest price is now uniquely determined. For $\underline{p} = \theta - \epsilon + \frac{1}{8}(1-c_a)^2$ the highest type has a strictly dominant strategy to buy. We show in the appendix that the firm never sets a price exceeding the highest type's intrinsic utility. This gives us a closed interval where the optimal price lies $p^* \in [\underline{p}, \theta + \epsilon]$. Effectively, p^* guarantees that the coordination game Γ has a unique NE. The idea is that both actions are played as strictly dominating strategies simultaneously under fulfilled expectations. Once this happens,

there is a unique marginal type who is indifferent between buying and not buying given by (6). The firm operates on the elastic section of the demand that is, we have $\theta - \epsilon \leq \overline{x}(q,p) \leq \theta + \epsilon$, with expectations being fulfilled $q^e = q$. In the unique surviving equilibrium, a fraction $q(p^*) \in]0,1]$ buy. Demand is given by

$$q = 1 - G\left(\overline{x}(q,p^*) \mid \theta\right). \tag{9}$$

We close this section by deriving the optimal price for a case with a unique equilibrium. There are two cases to consider: (i) $q(p^*) \geq \frac{1}{2}(1-c_a)$, and (ii) $q(p^*) < \frac{1}{2}(1-c_a)$. Numerical examples of both cases are provided in the appendix.

(i) Assume first that $q(p^*) \in \left[\frac{1}{2}(1-c_a), 1\right]$. We get demand from equation (9).

$$q = \frac{\theta + \epsilon - p}{2\epsilon - \frac{1}{8}(1-c_a)^2}. \tag{10}$$

Monopoly's profits are $\Pi = q(p)(p - c_f) + q(p)\pi_2^{**}(c_a)$. First order condition gives the optimal unit price

$$p^* = \frac{1}{2}\left(\theta + \epsilon + c_f - \pi_2^{**}\right). \tag{11}$$

Second order conditions are satisfied due to our assumption on low network externalities,

$$\frac{\partial^2 \Pi(p)}{\partial p^2} = -\frac{1}{\epsilon - \frac{1}{16}(1-c_a)^2} < 0.$$

(ii) Assume next that $q(p^*) \in \left[0, \frac{1}{2}(1-c_a)\right]$ in the second period. In this case, profits are $\Pi(p) = q(p)(p-c_f) + q(p)\pi_2^*(c_a, q)$. It is more convenient to solve for the indirect demand $p(q)$ from equation (9), and let the firm choose optimal quantity q^*. The first order condition is

$$2q^3 - 3q^2 - 2\left[2\epsilon - (1-c_a)\right]q + \theta + \epsilon - c_f = 0. \tag{12}$$

The second order condition requires

$$3q(q-1) < 2\epsilon - (1-c_a).$$

The resulting optimal price is higher in the case (ii) compared to the case (i), and demand is (by construction) lower in the case (ii). The second period price t^* is higher in the case (ii) due to low demand in the first period.

3.1 Equilibrium analysis

Whether heterogeneity between consumers is sufficient for uniqueness, can be rephrased as a question: Which factor is more important in consumers' decision making: standalone value (intrinsic utility) or interaction usage (network externalities)? A homogenous version of Γ, where consumers decide on the basis of perceived interaction usage, can be changed to a sufficiently heterogenous model by applying a mean-preserving spread on the consumer distribution.

Whenever the condition on sufficient heterogeneity is not satisfied, demand becomes indeterminate. At best we can assume that a particular NE emerges, and then characterise the firm's strategy in that equilibrium. We could do this for all possible equilibria. But we would lack any understanding why a particular equilibrium is selected. One method to select the equilibrium is to apply some anticipative cues à la Farrell & Katz (1998). For example, if consumers expect efficient coordination, coordination is efficient in the equilibrium and the firm sets a price corresponding to that NE. However, this approach is unsatisfactory. Essentially, the anticipative cues just give one, out of infinitely many possible explanations of the conditions which focalise a particular NE.

Equilibrium analysis is complicated since we have to keep track on the various possible states of θ, even with low network externalities. We need to distinguish between the cases $q(p^*) < \frac{1}{2}(1-c_a)$ and $q(p^*) \geq \frac{1}{2}(1-c_a)$.

Let us first analyse the case $q(p^*) \geq \frac{1}{2}(1-c_a)$. The optimal unit price is given by equation (11). Price increases as the heterogeneity between consumers increase

$$\frac{\partial p^*}{\partial \epsilon} = \frac{1}{2}.$$

When the optimal price is plugged back into (10), we get

$$q^* = \frac{\theta + \epsilon - c_f + \frac{1}{4}(1-c_a)^2}{4\left[\epsilon - \frac{1}{16}(1-c_a)^2\right]}. \tag{13}$$

Differentiation of (13) with respect to ϵ gives

$$\frac{\partial q^*}{\partial \epsilon} \underset{(<)}{>} 0 \Leftrightarrow \theta \underset{(>)}{<} c_f - \frac{5}{16}(1-c_a)^2.$$

The above rule defines a maximum state below which demand is increasing in consumer heterogeneity. When the cut-off state $\theta = c_f - \frac{5}{16}(1-c_a)^2$ is substituted in (13), we see that if demand satisfies $\max\left\{\frac{1}{4}, \frac{1}{2}(1-c_a)\right\} < q(p^*) \leq 1$, a marginal increase in consumer heterogeneity decreases demand, $\frac{\partial q(p^*)}{\partial \epsilon} < 0$. The marginal type is determined partly by the second period usage utility. Because consumers take into account their second period utility, an increase in heterogeneity has a stronger effect on demand than in the standard case where the marginal type is solely determined by the unit price. Demand increases in heterogeneity only if the state is low enough relative to unit costs adjusted with usage utility. A marginal increase in ϵ has a stronger effect on demand, the less heterogenous consumers are (that is the closer ϵ is to $\frac{1}{16}(1-c_a)^2$).

Profits in the case $q(p^*) \geq \frac{1}{2}(1-c_a)$ are

$$\Pi(p^*, t^*) = \frac{\left[\theta + \epsilon - c_f + \frac{1}{4}(1-c_a)^2\right]^2}{8\left[\epsilon - \frac{1}{16}(1-c_a)^2\right]}.$$

Consumer surplus is given by

$$\begin{aligned} S &= \frac{1}{2\epsilon}\int_{x=\overline{x}(q(p^*),p^*)}^{\theta+\epsilon} x + \frac{1}{8}(1-c_a)^2 q(p^*) - p^* dx \\ &= \left\{\frac{\theta + \epsilon - c_f + \frac{1}{4}(1-c_a)^2}{4\left[\epsilon - \frac{1}{16}(1-c_a)^2\right]}\right\}^2 \epsilon. \end{aligned}$$

A marginal change in ϵ has an ambiguous effect on profits and consumer surplus. There is a tendency for them to move in the same direction for marginal changes in ϵ. Profits increase when $\frac{\partial p^*}{\partial \epsilon}$ and $\frac{\partial q(p^*)}{\partial \epsilon}$ are both positive. In the cases where demand decreases as ϵ increases, profits tend to decrease when ϵ is close to its minimum (demand effect is stronger), and profits tend to increase as the absolute value of ϵ is large.

If costs are high so that the firm is constrained in the second period, $q(p^*) < \frac{1}{2}(1-c_a)$, a marginal change in consumer heterogeneity has a familiar effect on demand. Totally differentiating the first order condition (12), gives

$$\begin{aligned} \frac{dq^*}{d\epsilon} &> 0 \Leftrightarrow q^* < \min\left\{\frac{1}{4}, \frac{1}{2}(1-c_a)\right\} \\ \frac{dq^*}{d\epsilon} &< 0 \Leftrightarrow \frac{1}{4} < q^* < \frac{1}{2}(1-c_a). \end{aligned}$$

As a result, a positive marginal change in ϵ causes similar effects on demand as in the case where the firm is not constrained in the second period. Comparative statics for profits and consumer surplus are ambiguous and computatively complicated but present analogous tendencies.

4 Analysis with incomplete information

In the previous section we derived conditions for sufficient heterogeneity between consumers that guarantees uniqueness of equilibrium. We had a trade-off between the strength of network effects and heterogeneity. If we go for unique equilibrium, network externalities must be limited. This of course is perverse, if the model is designed to study network externalities.

Global games techniques require a different type of heterogeneity. We must have a *possibility* that the fundamental θ takes very low and very high values. Uniqueness does not hinge on the true heterogeneity between consumers (represented by the distribution $G(x \mid \theta)$). A limitation of players' capacity to observe information pays back in the relaxation of conditions for uniqueness.

The game remains otherwise unchanged from the perfect information case, except that consumers and the monopolist do not observe directly θ until at the end of period one. The actual value of θ is drawn again from the uniform distribution $F(\theta)$ with support $[-M, M]$. Consumers' observations of θ are blurred by noise. On the other hand, second period usage utility $\lambda(\alpha, t, q)$ is deterministic as the timing proves. The firm resorts to the prior on θ in its estimates. The consumers know that the firm is uninformed, which removes all possible information about θ that might otherwise be inferred from prices (p, t).

The consumer i gets an i.i.d. signal $x_i = \theta + \epsilon_i$, where ϵ_i is uniformly distributed on $[-\epsilon, \epsilon]$. The consumer who observes signal x gets an expected payoff gain from action $a = B$ versus $a = N$

$$v(x, q, p) = x + \lambda^*(q) - p. \tag{14}$$

Note that the signal enters directly the payoff function. Uncertainty over θ is thus a kind of horizontal differentiation. First period game is now a global game with private values. The payoff function is continuous in (x, q), even at the point $q = \frac{1}{2}(1 - c_a)$ where the indirect usage utility $\lambda^*(q)$ changes its shape. Payoffs are differentiable with respect to x everywhere, and the relation between payoffs and the signal x is positive, $\frac{\partial v(x,q,p)}{\partial x} > 0$. We also have strategic action complementarities in the sense $\frac{\partial v(x,q,p)}{\partial q} > 0$ (everywhere outside the cut-off point $q = \frac{1}{2}(1 - c_a)$). In addition, the payoffs satisfy the "strict Laplacian state monotonicity" assumption (see Morris & Shin 2003). Namely, there exists a unique $\widetilde{x}$ that solves $\int_{q=0}^{1} v(\widetilde{x}, q, p)\, dq = 0$. In sum, the payoff function (14) satisfies all the standard assumptions on strategic complementarities and continuity that global games require. The remaining condition we need to impose on payoffs in order to be able to use global games techniques is on dominance regions.

Condition 3 *Existence of dominance regions.*

(i) $\exists\ \underline{\theta} \in \left]-M, M\right[$ *so that* $v(x, q, p) < 0$ *for all* $q \in [0, 1]$ *and* $x \leq \underline{\theta}$.

(ii) $\exists\ \overline{\theta} \in \left]-M, M\right[$ *so that* $v(x, q, p) > 0$ *for all* $q \in [0, 1]$ *and* $x \geq \overline{\theta}$.

Note that the limits of the dominance regions are strictly contained within the prior distribution of θ. In other words, the prior of θ is sufficiently dispersed so that the boundaries of the distribution cause no trouble. Because the support of the prior is bounded, the Condition 3 is not trivially satisfied, as it would be if the support was from minus infinite to infinite (as in the case of e.g. normal distribution). Moreover, the dominance regions are endogenously determined due to monopoly pricing. In the appendix we derive sufficient bandwidth properties for the prior $F(\theta)$ and the conditional distribution of signals $G(x \mid \theta)$ so that Condition 3 holds.

Denote by Γ_{II} the incomplete information game with I consumers, pure actions $a \in \{B, N\}$, payoff (14), and where θ has a uniform prior, signals are i.i.d. and uniform, and where the distributions satisfy Condition 3. The coordination game Γ_{II} is supermodular since the action set is a compact subset of $\mathbb{R}$, and the payoff function (14) is continuous in its arguments and it has increasing differences in x. The implications of supermodularity are familiar: (i) a pure strategy Bayesian NE exists (not necessarily unique), (ii) the equilibria set has a smallest and largest element, and (iii) if there is a unique equilibrium, it is solvable by iterated deletion of strictly dominated strategies. Furthermore, due to action complementarity, the largest equilibrium element Pareto dominates other equilibria. This observation is, however, redundant as we will show that the game Γ_{II} has a unique Bayesian switching equilibrium denoted by Γ^*_{II}.

Proposition 4 *Let $\widetilde{x}$ be defined as the unique solution to $\int_{q=0}^{1} v(\widetilde{x}, q, p)\, dq = 0$. The game Γ_{II} has a unique switching strategy equilibrium Γ^*_{II} that survives iterated deletion of strictly dominated strategies. The unique equilibrium strategy satisfies $a(x) = N$ for all $x < \widetilde{x}$ and $a(x) = B$ for all $x > \widetilde{x}$.*

Proof. *In the appendix.* ■

We can now compute the marginal signal $\widetilde{x}$ that acts as the cut-off rule in the equilibrium strategy. Any observed signal above (below) this cut-off gives positive (negative) payoff for $a = B$. At the margin, when the consumer observes $x = \widetilde{x}$ exactly, he is indifferent between actions. His expectations about the fraction of people who play $a = B$ follows a uniform distribution on the unit interval. The marginal signal is given by

$$\int_{q=0}^{\frac{1}{2}(1-c_a)} \widetilde{x} + \frac{1}{2}q^3 - pdq + \int_{q=\frac{1}{2}(1-c_a)}^{1} \widetilde{x} + \frac{1}{8}(1-c_a)^2 q - pdq = 0, \tag{15}$$

where we have taken into account the cut-off point $t^* = \frac{1}{2}(1+c_a) \Rightarrow \alpha^*(t^*, q) = \frac{1}{2}(1-c_a)$, which is the point where the firm reaches its interior optimal usage fee. Integration of equation (15) gives

$$\widetilde{x} = p + \tau(c_a),$$

where $\tau(c_a) = \frac{1}{128}(1-c_a)^2\left[(1-c_a)^2 - 8\right]$ captures the expected second period usage utility.

When θ is the realisation of the fundamental, the proportion of consumers who get signals higher than $\widetilde{x}$ is $q = 1 - G(\widetilde{x} \mid \theta)$. They eventually buy the device. First period demand is

$$q(\theta, p) = \begin{cases} 1, \text{ if } \theta > \widetilde{x} + \epsilon \\ \frac{\theta+\epsilon-\tau(c_a)-p}{2\epsilon}, \text{ if } \widetilde{x} - \epsilon \leq \theta \leq \widetilde{x} + \epsilon \\ 0, \text{ if } \theta < \widetilde{x} - \epsilon \end{cases} . \tag{16}$$

Having defined the demand, we can turn to the pricing problem. Define the cut-off state $\widehat{\theta}$ as $q\left(\widehat{\theta}, p\right) = \frac{1}{2}(1-c_a)$. Whenever the true state is higher than $\widehat{\theta}$, the firm is not constrained in its second period problem, and its second period profits are $\Pi_2^* = q\pi_2^{**}(c_a)$. If the state is $\theta < \widehat{\theta}$, the optimal usage fee is at the corner solution $t^* = 1 - q$, and firm's second period profits are $\Pi_2^* = q\pi_2^*(c_a, q)$.

By assumption, the firm resorts to the prior in the first period, but it internalises the effect of p on the second period profits. Firm's expected profits are

$$\begin{aligned}\mathbb{E}(\Pi) &= \mathbb{E}(\Pi_1) + \mathbb{E}(\Pi_2) \\ &= \tfrac{1}{2M}\left\{\int_{\theta=\widetilde{x}-\epsilon}^{\widetilde{x}+\epsilon} q(\theta,p)(p-c_f)\,d\theta + \int_{\theta=\widetilde{x}+\epsilon}^{M} p - c_f d\theta + \right. \\ &\left. + \int_{\theta=\widetilde{x}-\epsilon}^{\widehat{\theta}} q(\theta,p)\,\pi_2^*(c_a,q)\,d\theta + \int_{\theta=\widehat{\theta}}^{\widetilde{x}+\epsilon} q(\theta,p)\,\pi_2^{**}(c_a)\,d\theta + \int_{\theta=\widetilde{x}+\epsilon}^{M} \pi_2^{**}(c_a)\,d\theta\right\}\end{aligned} \tag{17}$$

Two first integrals in equation (17) are associated with first period profits. Three last integrals capture the effect on second period profits. The tails of the prior's support do not cause trouble here, because consumers' behaviour does not change (due to dominance regions).

Maximisation of (17) gives the optimal price p^*. Denote the true state as θ^*, then the optimal price structure can be presented as in Proposition 5.

Proposition 5 *The optimal price structure is*

$$\begin{aligned} t^* &= \max\left\{\frac{1}{2}(1+c_a), 1 - q(\theta^*)\right\} \\ p^* &= \frac{1}{2}(M + c_f) - \frac{1}{2}\tau(c_a) - \frac{1}{8}(1-c_a)^2. \end{aligned}$$

Proof. *Derivation of the optimal price structure is in the appendix.* ■

The optimal prices are increasing in the usage cost,

$$\frac{dp^*}{dc_a} = \frac{1}{16}(1-c_a)\left[\frac{1}{4}(1-c_a)^2 + 3\right] \geq 0,$$

and

$$\frac{dt^*}{dc_a} = \begin{cases} \frac{1}{2},\ q(\theta^*) \geq \alpha\left(t^* = \frac{1}{2}(1+c_a)\right) \\ \frac{1-c_a}{4\epsilon}\left[\frac{5}{8} - \frac{1}{32}(1-c_a)^2\right],\ q(\theta^*) < \alpha\left(t^* = \frac{1}{2}(1+c_a)\right) \end{cases}$$

which is positive for $0 \leq c_a < 1$, and zero if $c_a = 1$ and the firm is at the corner solution in the second period.

The optimal unit price is increasing in unit production cost, $\frac{\partial p^*}{\partial c_f} > 0$. The usage fee is independent of production cost, as long as the firm is not constrained when setting t^*. If the realised demand binds the optimal usage fee, then we have $\frac{\partial t^*}{\partial c_f} = \frac{1}{4\epsilon} > 0$.

4.1 Role of uncertainty on profits and consumer surplus

Demand increases (decreases) in the precision of signals only if the revealed state θ is higher (lower) than the marginal signal. Why? When the precision of the signal is high, it tells the consumer that other people observe signals very close to the one he has observed. If the realisation of θ is below the marginal signal, and if signals are relatively accurate, then the consumer infers that most people do not buy. So, if $\theta < \widetilde{x}$ and we decrease the precision of signals ($d\epsilon > 0$), then a larger proportion of people may observe signals that are higher than the marginal signal. Therefore, a reduction in the precision of signals when $\theta < \widetilde{x}$ increases actual demand. Opposing, if $\theta > \widetilde{x}$ and signals are relatively precise, the consumer knows that other people observe signals close to true θ. If we reduce the precision of signals, a larger proportion of people observe signals that fall below the marginal signal. Therefore, a reduction in the precision of signals when $\theta > \widetilde{x}$ decreases demand. We summarise the above in Lemma 6.

Lemma 6 *Decrease in precision of signals decreases (increases) demand when true θ is above (below) the marginal signal,* $\frac{\partial q(\theta,p)}{\partial \epsilon} \underset{(>)}{<} 0 \Longleftrightarrow \theta \underset{(<)}{>} p + \tau(c_a)$, $\widetilde{x} - \epsilon \leq \theta \leq \widetilde{x} + \epsilon$.

Proof. *Proof follows directly from equation* (16)*, and thus omitted.* ■

The optimal unit price p^* is independent of uncertainty over signals. This is because we have assumed uniform distributions for the prior and signals. Resulting demand is linear, which renders first period profits $\mathbb{E}(\Pi_1(p^*))$ neutral with respect to ϵ. If the firm reaches the interior solution $t^* = \frac{1}{2}(1 + c_a)$ in the second period, also the usage fee is independent of any uncertainty. However, if the firm is pushed to the corner solution, the optimal usage fee is affected by uncertainty over signals. When the firm is constrained, we have $\left.\frac{\partial t^*}{\partial \epsilon}\right|_{q(\theta^*,p^*)<\frac{1}{2}(1-c_a)} = -\frac{\partial q(\theta^*,p^*)}{\partial \epsilon}$. The constrained optimal usage fee is higher than the interior solution. Because the firm is constrained with low values of θ, it is likely that $\frac{\partial q(\theta^*,p^*)}{\partial \epsilon} > 0$ holds. If the change in demand is positive $\frac{\partial q(\theta^*,p^*)}{\partial \epsilon} > 0$, the firm is able to lower its usage fee.

Because there is the possibility that the realised demand is low and the firm cannot charge the unconstrained optimal usage fee, expected second period profits are not independent of the precision of signals. The expected total profits are positively correlated with the precision of signals.

Proposition 7 *Increase in the precision of signals increases firm's profits*

$$\frac{\partial \mathbb{E}\left(\Pi\left(p^{*},t^{*}\right)\right)}{\partial \epsilon}=-\frac{\left(1-c_{a}\right)^{4}}{192M}\leq 0.$$

Proof. *In the appendix.* ■

If ϵ is increased marginally, in states that are below (above) the marginal signal demand increases (decreases). Whenever $\theta<\widetilde{x}-\epsilon$ firm's profits are zero. Therefore states that are above the marginal signal have a larger weight in expected profits. Therefore, the negative effect on demand is dominating. The negative effect on firm's profits comes from those (high) states where consumers are very confident on high sales. When uncertainty is increased, people have lower expectations on sales volumes, which induces lower sales and profits.

Consumer's expected surplus is

$$\begin{aligned}\mathbb{E}\left(S\right)=\frac{1}{4M\epsilon}&\left\{\int_{\theta=\widetilde{x}(p^{*})-\epsilon}^{\widehat{\theta}(p^{*})}\int_{x=\widetilde{x}(p^{*})}^{\theta+\epsilon}x+\frac{1}{2}q\left(\theta,p^{*}\right)^{3}-p^{*}dxd\theta+\right.\\&+\int_{\theta=\widehat{\theta}(p^{*})}^{\widetilde{x}(p^{*})+\epsilon}\int_{x=\widetilde{x}(p^{*})}^{\theta+\epsilon}x+\frac{1}{8}\left(1-c_{a}\right)^{2}q\left(\theta,p^{*}\right)-p^{*}dxd\theta+\\&\left.+\int_{\theta=\widetilde{x}(p^{*})+\epsilon}^{M}\int_{x=\theta-\epsilon}^{\theta+\epsilon}x+\frac{1}{8}\left(1-c_{a}\right)^{2}-p^{*}dxd\theta\right\}.\end{aligned}\tag{18}$$

To see the effect of a change in signals' precision, differentiate (18) with respect to ϵ. The sign of $\frac{\partial\mathbb{E}(S)}{\partial\epsilon}$ depends only on c_a and ϵ. Denote the solution to $\frac{\partial\mathbb{E}(S)}{\partial\epsilon}=0$ by $\overline{\epsilon}\left(c_a\right)$. We have plotted $\epsilon=\overline{\epsilon}\left(c_a\right)$ in figure (2). Above the curve, the derivative is positive, $\frac{\partial\mathbb{E}(S)}{\partial\epsilon}>0$, and below the curve we have $\frac{\partial\mathbb{E}(S)}{\partial\epsilon}<0$.

Proposition 8 *A decrease in signals precision ($d\epsilon>0$), induces:*

(i) *For relatively precise signals $\epsilon<\overline{\epsilon}\left(c_a\right)$, a decrease in expected consumer surplus.*

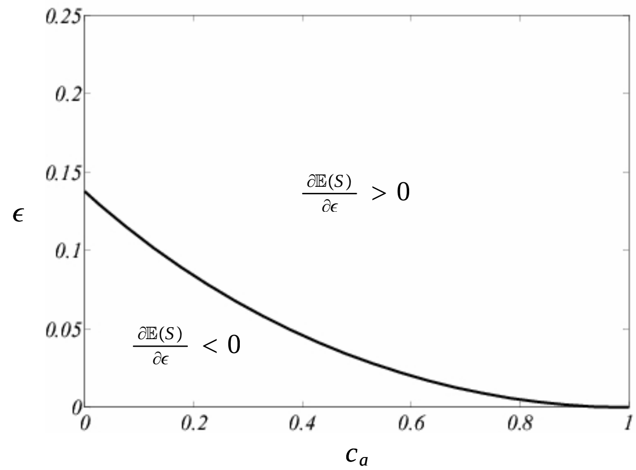

Figure 2: The Sign of $\frac{\partial \mathbb{E}(S)}{\partial \epsilon}$.

(ii) *For relatively imprecise signals* $\epsilon > \overline{\epsilon}(c_a)$, *an increase in expected consumer surplus.*

Unlike with profits, the absolute magnitude of ϵ plays a role in whether consumer surplus increases or decreases for marginal changes in the precision. When the signals are very precise (ϵ below the curve in figure (2)), the expected consumer surplus decreases as the precision of signals is marginally decreased ($d\epsilon > 0$). When signals are less precise (ϵ above the curve in figure (2)), consumer surplus is positively affected by a marginal increase in uncertainty. Expected surplus is affected via two effects. For a given θ, there is a change in the expected utility. There is also a change in the expectation of θ (that is, how the integration intervals change).

We can split the support of θ into sections and analyse what happens in each section. For a given θ, consumer's payoff is independent of ϵ if $\theta > \widetilde{x} + \epsilon$. Consumer's payoff decreases in ϵ when $\widetilde{x} < \theta < \widetilde{x} + \epsilon$. Payoff increases in ϵ, when the state is $\widetilde{x} - \epsilon < \theta < \widetilde{x}$. Payoff is zero for $\theta < \widetilde{x} - \epsilon$. When ϵ increases, the highest segment $[\widetilde{x} + \epsilon, M]$ shortens. Segment $[\widetilde{x}, \widetilde{x} + \epsilon]$ grows. For $c_a > 0$, segment $\left[\widehat{\theta}, \widetilde{x}\right]$ grows, but for $c_a = 0$, the cut-off points coincide $\widehat{\theta} = \widetilde{x}$. The segment $\left[\widetilde{x} - \epsilon, \widehat{\theta}\right]$ grows for $0 < c_a < 1$, but for $c_a = 0$ we have $\widehat{\theta} = \widetilde{x}$ and the segment is unmodified, for $c_a = 1$, we have $\widehat{\theta} = \widetilde{x} - \epsilon$ so that the segment is of zero length. Finally, when ϵ increases, the segment $[-M, \widetilde{x} - \epsilon]$, where payoffs are zero, shortens.

The negative effect on surplus is foremost associated with the very high states ($\theta > \widetilde{x} + \epsilon$), where expected consumer surplus unambiguously reduces as ϵ increases. This is the segment where consumers are confident on high sales. The negative effect is stronger the smaller ϵ and c_a are, which shows up in that the total effect turns negative in the area $\epsilon < \overline{\epsilon}(c_a)$. For lower values of θ, there is a mixture of positive and negative effects. The summation of effects returns the result illustrated in figure (2).

We have a minimum for $\mathbb{E}(S(\epsilon))$ given by $\overline{\epsilon}(c_a)$, captured in the curve in figure (2). If signals are extremely precise ($\epsilon \to 0$), so that consumers are homogeneous, the consumer benefits from the knowledge that other people are like him. In this case, network externalities have important role in decision making. When we reduce the precision of signals, the little uncertainty about other people hurts. However, when the precision of signals drops to a relatively low level ($\epsilon > \overline{\epsilon}(c_a)$), higher uncertainty is of good. Why? Low precision is analogous to heterogeneity between consumers. If signals are imprecise, the consumer knows that there is a large variance in the perception of the true intrinsic value of the product within the population, and knows that other people know that everybody is equally uninformed. The consumer is then likely to base his buying decision on the intrinsic utility, rather than on the expected the behaviour of other people. In this case, the consumers benefit from further knowledge ($d\epsilon > 0$) about the fact that they can base their decisions on their private values. Similarly, if signals are accurate ($\epsilon < \overline{\epsilon}(c_a)$), further information ($d\epsilon < 0$) on that perceived network externalities are driving everybody else's decisions increases expected surplus.

5 Comparison

In this section we discuss the differences between perfect and incomplete information regimes. We focus on the perfect information case where (i) network externalities are sufficiently low to guarantee a unique equilibrium, and (ii) the firm is not constrained in the second period giving a higher monopoly price compared with the constrained case. Let us restate the optimal prices under perfect and incomplete information for reference

$$
\begin{aligned}
p_{PI}^{*} &= \frac{1}{2}\left(\theta+\epsilon\right)+\frac{1}{2}c_{f}-\frac{1}{8}\left(1-c_{a}\right)^{2} \\
p_{II}^{*} &= \frac{1}{2}M+\frac{1}{2}c_{f}-\frac{1}{8}\left(1-c_{a}\right)^{2}-\frac{1}{2}\tau\left(c_{a}\right).
\end{aligned}
$$

The term $-\frac{1}{8}\left(1-c_{a}\right)^{2}$, present in both price equations, is the effect from second period profits. The firm takes into account that high first period price reduces second period profits. This effect is eliminated if we introduce perfect competition in the second period, so that usage fee is $t=c_{a}$. The optimal first period monopoly price under incomplete information when the second period is characterised by perfect competition is

$$
p_{C}^{*}=\frac{1}{2}\left(M+c_{f}-\tau_{C}\left(c_{a}\right)\right),
$$

where $\tau_{C}\left(c_{a}\right)\leq\tau\left(c_{a}\right)\leq 0$. Derivation of p_{C}^{*} is in the appendix.

Prices p_{PI}^{*} and p_{II}^{*} diverge in two respects. First, because the firm observes nothing under incomplete information, it takes expectations on the consumer distribution. So, we have M replacing $\theta+\epsilon$ in the prices. Consequently, the price tends to be higher under incomplete information. Only when the realised state θ is close to the upper limit M so that there are types above M, price is higher under perfect information. Under incomplete information, the unit price is independent of the term measuring heterogeneity ϵ (i.e. independent of uncertainty), which is in contrast to the perfect information case, in which the firm increases the price for a marginal increase in consumer heterogeneity.

The second, more interesting, difference is the term $-\frac{1}{2}\tau\left(c_{a}\right)\geq 0$, which captures the firm's (accurate) perception on what are *consumers'* expectations on the second period usage utility. Because under perfect information, all players (including the firm) observe perfectly how much usage utility consumers get in the second period, the effect is neutralised in the unit price. There is a one-to-one relationship between expected usage utility $(q\widehat{\lambda}^{*}\left(q\right))$ and first period price $p=\theta+\epsilon-q\left(2\epsilon-\widehat{\lambda}^{*}\left(q\right)\right)$. Under incomplete information, consumers' expectations are "fixed", so there is a (potential) gap between expected and actual usage utility. This gap induces a safer pricing strategy: the firm prices high in the first period, before consumers learn the true state, at the expense of more uncertain second period profits. When the firm is uncertain about second period usage utility, it adjusts its price upwards. This effect is aggravated, when the second period is characterised by perfect competition (with incomplete information). We have $-\frac{1}{2}\tau_C(c_a) \geq -\frac{1}{2}\tau(c_a)$, so the monopoly has incentives to set even higher price. It does not have any incentives to insure second period profits by setting a low first period price. Demand, however, is higher for the (second period) competition case than for the two-period monopoly, $q(p_C^*) > q(p^*)$, because the monopoly limits supply in the second period.

The expectations mechanism affects the realised demand, and we cannot tell unambiguously, whether demand is higher under perfect or incomplete information. Numerical simulations that we have carried out tend to result in higher demand under perfect information.

The term determining real heterogeneity between consumers (and measuring uncertainty), ϵ, has an important role for coordination on a unique equilibrium under perfect information. This role is taken away if consumers' valuations are private information (but correlated). Even the smallest amount of uncertainty is sufficient to result in a unique equilibrium, whereas we had an explicit rule for minimum heterogeneity under perfect information.

If $\epsilon < \frac{1}{16}(1-c_a)^2$ under perfect information, the game is inflicted by the multiple equilibria problem. If coordination failure is imminent, firm's preferences over heterogeneity differ under perfect and incomplete information regimes. Let the network externalities be high and information perfect. If the market sentiment is pessimistic, so that coordination is prone to fail, the firm may prefer more heterogeneity between consumers. Higher heterogeneity facilitates coordination on the efficient NE. Under super pessimistic expectations, the firm prefers high heterogeneity which would support a (larger) unique equilibrium. If we maintain high network externalities, but impose incomplete information, coordination is unaffected by the real heterogeneity between consumers. Moreover, we know that the firm's profits increase as ϵ decreases, $\frac{\partial \mathbb{E}(\Pi(p^*, t^*))}{\partial \epsilon} < 0$. So it prefers little heterogeneity.

Perfect information is a strong condition. For ordinary consumable goods, the assumption on perfect information does not (necessarily) create problems. But, in problems of coordination, even marginal difference between perfect information and almost perfect information can produce strikingly different outcomes (see the discussion in Morris 2002). We have presented a model of adoption of a new device that enables efficient interaction between people. The analysis shows that whether we impose perfect or incomplete information, the predictions of the model differ. The analytical easiness of the incomplete information model compared with the perfect information case, favours the limitation of people's observation capabilities. More importantly, for a novel product, incomplete information regime also characterises the real world more accurately. Just think about how we are more capable of saying how much utility a fax machine or e-mail client software yields to other people today than, say, we were twenty years ago. As the product matures, information becomes more accessible.

6 Concluding remarks

We have analysed a market for network goods. A monopolist launches a device that enables efficient interaction between people. Hence, consumers face a coordination problem whether to switch to using the new device or to stick with the prevailing interaction systems. This kind of coordination game has multiple Nash equilibria under perfect information and homogenous players. We have done a comprehensive analysis how uniqueness of equilibrium can be reached in our model. The interpretation we have given for the necessary conditions for uniqueness apply to network models in general. Uniqueness of equilibrium under perfect information requires high consumer heterogeneity. Adversely, we must limit the role of network externalities in consumers' buying decision making. Under incomplete information, uniqueness of equilibrium arises endogenously, as long as the prior distribution of the underlying economic fundamental is sufficiently dispersed.

The key to uniqueness is the same in both informational regimes. When one group of people play "Buy" as a strictly dominant strategy, at the same time as another group play "Not Buy" as a strictly dominant strategy, the resulting equilibrium is unique. Both information regimes required some level of heterogeneity, but the type of heterogeneity is different in the two cases. Under perfect information, heterogeneity between consumers had to be real. Under incomplete information, uniqueness does not hinge on the real heterogeneity between people, which can be minimal. Instead of real heterogeneity, we needed to raise a *possibility* that the fundamental value of the product can be very low or very high. Hence, the cost of heavier informational system imposed by global games pays back in less restricting assumptions on model parameters.

Uniqueness of the equilibrium allowed us to carry out comparative statics analysis. The monopoly price is independent of consumer heterogeneity under incomplete information, but it is increasing in heterogeneity under perfect information. The optimal price tends to be higher under incomplete information.

A marginal change in consumer heterogeneity has an ambiguous effect on profits and consumer surplus under perfect information. The effect is even more ambiguous if network externalities are strong, so that we have more than one Nash equilibrium in the coordination game. Under incomplete information, firm's expected profits increase as the precision of signals improves (i.e. heterogeneity between consumers is lowered). The effect on expected consumer surplus depends on the absolute level of the signal's precision. The expected consumer surplus increases if the marginal change in the precision of signals is in-line with the way consumers base their buying decisions. If signals are precise (imprecise), further improvement (reduction) in accuracy raises surplus. In this sense, better agreement on the factor that drives decision making among the consumers is of good.

Perfect and incomplete information regimes yield strongly different predictions about adoption of a new interaction device. On the one hand, under perfect information, the coordination failure is a probable scenario. If the main selling argument is based on interaction usage, the perfect information variant is in trouble in explaining which equilibrium is the most probable one. On the other hand, under incomplete information, there is no coordination failure. We have argued that for novel goods the private information case is more realistic characterisation of the world. So, facing castigation in the board room of Electro Fads ltd.[6] , the Sales and Marketing Director cannot hide behind "bad market sentiments", but should blame (justly) the Product Development Director for the low quality of the device.

The incomplete information case offers a number of possible extensions that would be interesting to analyse. We have used fairly specific distributions, and utility functions, that could be generalised. The uniqueness result also allows analysis on strategic investments that has been previously obstructed by the multiplicity problem in network models. We assumed that consumers know their needs better than the firm. It would be interesting to allow the firm to observe something more than nothing. It could then use prices to manipulate consumers' perceptions of the value of the fundamental.

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[6] Fictitious firm, any resemblance to real firms, solvent or bankrupt, is coincidental.

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8 Appendix

8.1 Proofs and numerical examples for perfect information regime

Multiple equilibria under high network externalities. We show that when network externalities are high, $\epsilon < \frac{1}{16}(1-c_a)^2$, (i) under efficient coordination (corresponding to the maximal NE of the game Γ), there exists a profits maximising price which exceeds the highest type's intrinsic utility; (ii) any profits maximising price p^* associated with efficient coordination induces multiple equilibria, and therefore under a coordination failure, the firm chooses a lower price than under efficient coordination. Prohibitive state-cost pairs $\left(\theta^-, c_f^+\right)$ defined in (8) are categorically ruled out.

(i) When coordination is efficient, we have a minimum price that the firm will ever charge

$$\underline{p} = \theta - \epsilon + \frac{1}{8}(1-c_a)^2,$$

which corresponds to the price that leaves the lowest type indifferent between buying and not when everybody else buys. By the assumption of high network externalities, we have $\underline{p} > \theta + \epsilon$. There is also an upper boundary price $\overline{p} = \theta + \epsilon + \lambda^*(q)$, above which it becomes dominant strategy for everyone not to buy. Note that the upper boundary price is less than what leaves the highest type indifferent between buying and not buying when everybody else buys. Hence, if there is a profits maximising price under high network externalities, it belongs to the closed interval $p^* \in \left[\underline{p}, \overline{p}\right]$. Demand $q(p)$ given by (7) is continuous in p in the interval $\left[\underline{p}, \overline{p}\right]$ due to continuity of types x and continuity of $\lambda^*(q)$ in q. Because demand is zero at the upper extreme, $q(\overline{p}) = 0$, the firm makes zero profits $\Pi(\overline{p}) = 0$. At the low extreme, demand equals one, $q\left(\underline{p}\right) = 1$, but profits can be anything depending on cost parameters and the realisation of the state. Firm's profits are

$$\Pi(p) = \begin{cases} q(p)(p - c_f) + q(p)\pi_2^{**}(c_a), \text{ if } q(p^*) \geq \frac{1}{2}(1 - c_a) \\ q(p)(p - c_f) + q(p)\pi_2^{*}(c_a, q), \text{ if } q(p^*) < \frac{1}{2}(1 - c_a) \end{cases} . \tag{19}$$

Due to continuity of demand in $p \in [\underline{p}, \overline{p}]$, profits (19) are continuous, even at the cut-off point $q(p) = \frac{1}{2}(1 - c_a)$. By Weierstrass' Theorem, there exists $p^* \in \arg\max\{\Pi(p)\}$ in the interval $[\underline{p}, \overline{p}]$. The optimal price may be an interior solution or a corner solution.

(ii) From part (i), we know that the optimal price is bounded in the region $p^* \in [\underline{p}, \overline{p}]$ when coordination is efficient. Importantly, the lower boundary price exceeds the highest type's intrinsic utility, $\underline{p} > \theta + \epsilon$. Consider next the case where the firm sets price $p^* \in [\underline{p}, \overline{p}]$ expecting efficient coordination, but consumers' expectations are super pessimistic, $\mathbb{E}(q) = 0$. But, we have $p^* > \theta + \epsilon$, so all consumers now expect negative payoffs from buying, and therefore no-one buys. Any price $p^* \in [\underline{p}, \overline{p}]$ supports both an efficient coordination NE where a positive proportion of consumers buy, and a "no-one buys" NE. In equilibrium, the firm, of course, knows to which NE consumers coordinate on and adjusts its price accordingly. Under total coordination failure, $\mathbb{E}(q) = 0$, the highest price that guarantees that the "no-one buys" equilibrium is evaded is $p = \theta + \epsilon$, at which point the highest type becomes indifferent between buying and not when no-one else buys. Consequently, the firm sets a price $p^* < \theta + \epsilon$ and makes positive profits.

From (i) - (ii) we get the result that under high network externalities, there are always multiple equilibria in the consumers' coordination subgame Γ parameterised by price. It is worth noting that efficient coordination NE and super pessimistic expectations NE correspond to rational expectations, $\mathbb{E}(q) = q$. We need some exogenous cues à la Farrell & Katz (1998) to explain which NE emerges. For each equilibrium expectation we obtain a specific demand function. Optimal monopoly price is different in different NE. Efficient coordination supports the highest optimal price which is set above the highest type's intrinsic utility. Under a coordination failure, the monopoly incorporates the pessimistic expectations and adjusts its price downwards. At the extreme, under total coordination failure, the firm must set a price below the highest type's valuation.

■

Unique equilibrium under low network externalities. We give proof that under low network externalities, $\epsilon > \frac{1}{16}(1-c_a)^2$, the optimal price is always bounded within $p^* \in \left[\theta - \epsilon + \frac{1}{8}(1-c_a), \theta + \epsilon\right]$. This price yields a unique equilibrium. The proof is constructed in two steps. First, we show that the optimal price is always bounded from below. Second, we show that the price is also bounded from above. Thanks to continuity properties we have a price that maximises profits.

Let us start by restating the indifferent type, when consumer expectations are $\mathbb{E}(q) \equiv q^e$ (equation (6))

$$\overline{x}(q^e, p) = p - \lambda^*(q^e),$$

which gives the corresponding demand schedule (equation (7))

$$q(p, q^e) = \begin{cases} 0, & \text{if } \overline{x}(q^e, p) > \theta + \epsilon \\ \frac{\theta + \epsilon + \lambda^*(q^e) - p}{2\epsilon}, & \text{if } \theta - \epsilon \leq \overline{x}(q^e, p) \leq \theta + \epsilon \\ 1, & \text{if } \overline{x}(q^e, p) < \theta - \epsilon \end{cases}.$$

In equilibrium, expectations are fulfilled, so that $q^e = q$. Prohibitive state-cost pairs $\left(\theta^-, c_f^+\right)$ defined in (8) are categorically ruled out.

(i) The lowest price the firm will ever charge is $\underline{p} = \theta - \epsilon + \frac{1}{8}(1-c_a)$. This price leaves the lowest type indifferent between buying and not when everybody else buys. By the assumption of low network externalities, for price $\underline{p} = \theta - \epsilon + \frac{1}{8}(1-c_a)$, the highest type has a strictly dominant strategy to buy. Since the highest type has strictly dominant strategy to buy, even under super pessimistic expectations ($\mathbb{E}(q) = 0$) the firm makes positive sales with price $\underline{p}$.

(ii) Next we prove that the optimal price does not exceed the highest type's valuation, $p^* \leq \theta + \epsilon$. Start by assuming $q(p^*) \geq \frac{1}{2}(1-c_a)$. In this case, the demand corresponding to fulfilled expectations is given by

$$q(p) = \begin{cases} \frac{\theta+\epsilon-p}{2\left[\epsilon-\frac{1}{16}(1-c_a)^2\right]}, & \text{if } \theta-\epsilon+\frac{1}{8}(1-c_a)^2 \le p \le \theta+\epsilon-(1-c_a)\left[\epsilon-\frac{1}{16}(1-c_a)^2\right] \\ 1, & \text{if } p < \theta-\epsilon+\frac{1}{8}(1-c_a)^2 \end{cases}.$$

We have $\frac{\partial q(p)}{\partial p} < 0$ in the range where the demand is elastic. So, the highest feasible price is obtained when demand is $q = \frac{1}{2}(1-c_a)$. This price is $p = \theta+\epsilon-(1-c_a)\left[\epsilon-\frac{1}{16}(1-c_a)^2\right] \le \theta+\epsilon$ with equality at $c_a = 1$.

Assume next that $q(p^*) < \frac{1}{2}(1-c_a)$ so that the firm is in the corner solution in the second period. It is now more convenient to solve for the inverse demand function. In equilibrium, we have the price

$$p(q) = \theta+\epsilon-q\left(2\epsilon-\frac{1}{2}q^2\right), \tag{20}$$

which can be increasing in $q \in \left[0, \frac{1}{2}(1-c_a)\right]$. The firm maximises profits by choosing quantity $q^* \in \left[0, \frac{1}{2}(1-c_a)\right]$. Since we are interested in the possibility of the case $p(q^*) > \theta+\epsilon$, the term in parenthesis in (20) should be negative. So, we require that $\epsilon < \frac{1}{4}q^2$ holds. As we combine this condition with the initial assumption on low network externalities, we have a range within the heterogeneity parameter must be strictly bounded $\frac{1}{16}(1-c_a)^2 < \epsilon < \frac{1}{4}q^2$. This condition is the least binding when demand q is at maximum. But, our assumption $q^* < \frac{1}{2}(1-c_a)$ gives the maximal consistent equilibrium demand level. Once this level is plugged into the condition, we end up with $\frac{1}{16}(1-c_a)^2 < \epsilon < \frac{1}{16}(1-c_a)^2$, which cannot hold. Hence, if we force $\epsilon < \frac{1}{4}q^2$ to hold, we violate $\frac{1}{16}(1-c_a)^2 < \epsilon$, and vice versa. Consequently, the price remains bounded from above $p(q) \le \theta+\epsilon$. Note that price is continuous in $q \in [0,1]$. If we plug $q = \frac{1}{2}(1-c_a)$ in (20), we get $p = \theta+\epsilon-(1-c_a)\left[\epsilon-\frac{1}{16}(1-c_a)^2\right]$.

In (i) - (ii) we have established that the firm sets a price $p^* \in \left[\theta-\epsilon+\frac{1}{8}(1-c_a), \theta+\epsilon\right]$. Firm's profits are

$$\Pi(p) = \begin{cases} q(p)(p-c_f)+q(p)\pi_2^{**}(c_a), & \text{if } q(p) \ge \frac{1}{2}(1-c_a) \\ q(p)(p-c_f)+q(p)\pi_2^{*}(c_a,q), & \text{if } q(p) < \frac{1}{2}(1-c_a) \end{cases}.$$

Because profits are continuous in p there exists a profits maximising price $p^* \in \left[\theta - \epsilon + \frac{1}{8}(1-c_a), \theta + \epsilon\right]$ by Weierstrass' Theorem.

For price p^*, the lowest type gets zero payoff at maximum

$$v(\theta - \epsilon, q, p^*) \leq 0 \quad \forall q \in [0,1]. \tag{21}$$

At the same time, the highest type always gets at least zero payoff

$$v(\theta + \epsilon, q, p^*) \geq 0 \quad \forall q \in [0,1]. \tag{22}$$

Inequalities (21) and (22) establish (weak) dominance regions. Despite that at the boundaries $p \in \left\{\theta - \epsilon + \frac{1}{8}(1-c_a), \theta + \epsilon\right\}$ everybody may play the same action, the equilibrium is unique. When the price is $p^* = \theta - \epsilon + \frac{1}{8}(1-c_a)$ the lowest type is indifferent between buying and not, so the indeterminacy is limited to him only. Similarly, when the price is $p^* = \theta + \epsilon$, the highest type is indifferent between buying and not, and he is the only one whose action is indeterminate. Since these are marginal cases, we can ignore them. As a result, the equilibrium is unique. ■

Cases $q(p^*) < \alpha^*$ and $q(p^*) > \alpha^*$ under sufficient heterogeneity for uniqueness. We provide two examples that illustrate how the monopolist sets price so that it is (i) constrained in the second period, and (ii) reaches interior optimum in the second period. Assume that consumers are sufficiently heterogenous, or in other words, network externalities are relatively low, $\epsilon > \frac{1}{16}(1-c_a)^2$ in order to a have a unique equilibrium.

(i) Let $c_a = \frac{1}{4}$, $c_f = \frac{12}{10}$, $\theta = 1$, and $\epsilon = \frac{1}{2}$. Start by assuming $q^* < \frac{1}{2}(1-c_a) = \frac{3}{8}$. Demand is obtained from equation (12), $q^* \approx 0.2614$, which is consistent with our initial assumption. The corresponding price is $p(q^*) \approx 1.2475$. We can check that the equilibrium is unique. Even with super pessimistic expectations the highest type has a strictly dominant strategy to buy, $v(\theta + \epsilon, q^e = 0, p^*) \approx 0.2525$. The lowest type has a strictly dominant strategy not to buy, $v(\theta - \epsilon, q^e = 1, p^*) \approx -0.6772$ even under super optimistic expectations. The indifferent type is located at $\overline{x} \approx 1.2386$. Firm's profits are $\Pi(q^*, t^*) = 0.0458$, and it makes profits in both periods.

(ii) Let $c_a = \frac{1}{4}$, $c_f = \frac{1}{2}$, $\theta = 1$, and $\epsilon = \frac{1}{2}$. Start by assuming $q(p^*) > \frac{1}{2}(1 - c_a) = \frac{3}{8}$. Optimal price is given by equation (11), $p^* = \frac{119}{128}$. Plugging p^* in equation (10), we get the first period demand $q(p^*) = \frac{73}{119} \approx 0.6134$, which is consistent with our initial assumption. We can check that the equilibrium is unique. The highest type has a strictly dominant strategy to buy, $v(\theta + \epsilon, q^e = 0, p^*) = \frac{73}{128}$. The lowest type has a strictly dominant strategy not to buy, $v(\theta - \epsilon, q^e = 1, p^*) = -\frac{23}{64}$. The indifferent type is located at $\overline{x} = \frac{211}{238}$. Firm makes profits in both period. Total profits are $\Pi(p^*, t^*) \approx 0.3499$.

■

8.2 Proofs for incomplete information regime

Sufficient bandwidth in the incomplete information regime (Condition 3). The dominance regions must exist for all consumer expectations. We show that as M is sufficiently large, lower and upper dominance regions coexist.

(i) Let us first consider the upper dominance region: $\exists\, \overline{\theta} \in]-M, M[$ so that $v(x, q, p) > 0$ for all $q \in [0, 1]$ and $x \geq \overline{\theta}$. Assume that consumers are optimistic and expect full coverage $q^e = 1$. Because consumers expect $q^e = 1$, they also expect second period usage utility $\lambda^*(q^e) = \frac{1}{8}(1 - c_a)^2$. The consumer who observes x and has expectations $q^e = 1$, gets expected payoff gain $v = x + \frac{1}{8}(1 - c_a)^2 - p$. Because $v(x, q, p)$ is strictly increasing in x, we get a marginal type $\overline{x}_{q^e=1} = p - \frac{1}{8}(1 - c_a)^2$ who is indifferent between buying and not buying. The true demand schedule under expectations $q^e = 1$ is

$$q(p) = \begin{cases} 0, & \text{if } \overline{x}_{q^e=1} > \theta + \epsilon \\ \frac{\theta + \epsilon + \frac{1}{8}(1 - c_a)^2 - p}{2\epsilon}, & \text{if } \theta - \epsilon \leq \overline{x}_{q^e=1} \leq \theta + \epsilon \\ 1, & \text{if } \overline{x}_{q^e=1} < \theta - \epsilon \end{cases} \tag{23}$$

Demand (23) corresponds to the most "optimistic" expectations, thus it supports the highest monopoly price. Define the cut-off state $\widehat{\theta}_{q^e=1}$ below which the monopoly is constrained in the second period and above the firm reaches the interior solution. The monopoly's expected profits, when consumer expectations are optimistic $q^e = 1$, are

$$\begin{aligned} \mathbb{E}(\Pi) &= \frac{1}{2M}\left\{\int_{\overline{x}_{q^e=1}-\epsilon}^{\widehat{\theta}_{q^e=1}} q(p)(p-c_f) + q(p)\pi_2^* d\theta + \right. \\ &\quad + \int_{\widehat{\theta}_{q^e=1}}^{\overline{x}_{q^e=1}+\epsilon} q(p)(p-c_f) + q(p)\pi_2^{**} d\theta + \\ &\quad \left. + \int_{\overline{x}_{q^e=1}+\epsilon}^{M} p - c_f + \pi_2^{**} d\theta \right\}. \end{aligned} \tag{24}$$

Optimisation of (24) gives price

$$p^*_{q^e=1} = \frac{1}{2}\left[M + c_f - \frac{1}{8}(1-c_a)^2\right].$$

The second order conditions are satisfied, $\frac{\partial^2 \mathbb{E}(\Pi)}{\partial p^2} = -\frac{1}{M} < 0$. Given the price $p^*_{q^e=1}$, the highest type must have a strictly dominant strategy to buy, even if no-one else buys, $M + \epsilon - p^*_{q^e=1} > 0$, where we have used $\lambda^*(q^e = 0) = 0$, which gives the following condition on the bandwidth of θ's and x's distribution

$$M + 2\epsilon > c_f - \frac{1}{8}(1-c_a)^2. \tag{25}$$

(ii) A similar line of reasoning must apply to the lower dominance region, $\exists\, \underline{\theta} \in \left]-M, M\right[$ so that $v(x,q,p) < 0$ for all $q \in [0,1]$ and $x \le \underline{\theta}$. Now, we look for the optimal price corresponding to the most "pessimistic" expectations $q^e = 0$. This price is the lowest price the firm will ever set. We skip the derivation of the true demand schedule corresponding to expectations $q^e = 0$, and the calculation of the respective optimal price. The procedures are identical to those explained in part (i). Given the optimal price $p^*_{q^e=0} = \frac{1}{2}\left[M + c_f - \frac{1}{4}(1-c_a)^2\right]$ corresponding to the most pessimistic expectations, the lowest type must have a strictly dominant strategy not to buy, even if everybody else buys, $-M - \epsilon + \frac{1}{8}(1-c_a)^2 - p^*_{q^e=0} < 0$. Note that we need to apply $\lambda^*(q^e = 1) = \frac{1}{8}(1-c_a)^2$. The second order conditions are satisfied, $\frac{\partial^2 \mathbb{E}(\Pi)}{\partial p^2} = -\frac{1}{M} < 0$. As a result, the following requirement for distribution bandwidths is obtained

$$-M-\frac{2}{3}\epsilon<\frac{1}{3}\left[c_f-\frac{1}{2}\left(1-c_a\right)^2\right]. \tag{26}$$

The requirements (25) and (26) are satisfied simultaneously when we expand the support of $F(\theta)$ by increasing M sufficiently. When M is sufficiently large, the heterogeneity ϵ can afford to go to zero at the limit. ■

Proof of Proposition 4. The proof follows Morris & Shin (2003). We look for a switching equilibrium with a unique switching point. The switching strategy with a cut-off point k is a function

$$s(x)=\begin{cases} N, \text{ if } x<k \\ B, \text{ if } x>k, \end{cases}$$

where the consumer is indifferent between actions at the switching point k.

When a consumer has observed signal x, he places (conditional) density $h(\theta \mid x)$ on any state θ. Denote $f(\theta)=\frac{1}{2M}$ as the unconditional density of the uniformly distributed underlying fundamental. When a state θ is realised, signals are also uniformly distributed, so that the density of signals is $g(x \mid \theta)=\frac{1}{2\epsilon}$. The conditional density of θ, when signal x has been observed, is

$$\begin{aligned} h(\theta \mid x) &= \begin{cases} \frac{f(\theta)g(x|\theta)}{\int_{\theta=x-\epsilon}^{x+\epsilon} f(\theta)g(x|\theta)d\theta}, \text{ if } x-\epsilon\leq\theta\leq x+\epsilon \\ 0, \text{ otherwise} \end{cases} \\ &= \begin{cases} \frac{1}{2\epsilon}, \text{ if } x-\epsilon\leq\theta\leq x+\epsilon \\ 0, \text{ otherwise} \end{cases}. \end{aligned}$$

The probability that a signal higher than k is observed when the state is θ is $\mu(k)=1-G(k \mid \theta)$, where G is the uniform conditional distribution function of density $g(x \mid \theta)$ on the support $[\theta-\epsilon, \theta+\epsilon]$. The probability $\mu(k)$ is decreasing in k and increasing in θ. By the law of large numbers, $\mu(k)$ equals the probability that fraction $\mu(k)$ of (other) people at maximum get signals higher than k. The expected payoff gain from choosing action $a=B$ for a consumer who has observed signal x and knows that all other consumers will choose action $a=N$ if they observe signals less than k can be written as

$$\begin{aligned}\mathbb{E}\left[v\left(x,k,p\right)\right] &= \int_{\theta=x-\epsilon}^{x+\epsilon} h\left(\theta \mid x\right) v\left(x,\mu\left(k\right),p\right) d\theta \\ &= \tfrac{1}{2\epsilon}\int_{\theta=x-\epsilon}^{x+\epsilon} x+\lambda^{*}\left(\mu\left(k\right),t^{*}\right)-pd\theta,\end{aligned} \tag{27}$$

where $\lambda^{*}\left(\mu\left(k\right),t^{*}\right)$ is the indirect usage utility, as defined in equation (4). The expected payoff (27) is continuous in x and in k despite the kink in the demand for usage.

Next we show that the expected utility $\mathbb{E}\left[v\left(x,k,p\right)\right]$ is strictly increasing in x and strictly decreasing in k everywhere, i.e. it presents increasing differences and strict strategic complementarities.

Even though usage is always at optimal level from the consumers' point of view, we need to consider two cases in order to prove $\frac{\partial\mathbb{E}[v(x,k,p)]}{\partial x} > 0$ and $\frac{\partial\mathbb{E}[v(x,k,p)]}{\partial k} < 0$. First, when the firm reaches its interior optimal fee $t^{*} = \frac{1}{2}\left(1+c_a\right)$, the expected payoff (27) is $\mathbb{E}\left[v\left(x,k,p\right)\right] = \frac{1}{2\epsilon}\int_{\theta=x-\epsilon}^{x+\epsilon} x+\frac{1}{8}\left(1-c_a\right)^2\mu\left(k\right)-pd\theta$. In the second case, the firm is constrained to $t^{*}=1-\mu\left(k\right)$, and the expected payoff (27) is $\mathbb{E}\left[v\left(x,k,p\right)\right] = \frac{1}{2\epsilon}\int_{\theta=x-\epsilon}^{x+\epsilon} x+\frac{1}{2}\mu\left(k\right)^3-pd\theta$.

When consumers are at their optimum, and the firm at the interior solution, we have

$$\frac{\partial\mathbb{E}\left[v\left(x,k,p\right)\right]}{\partial x} = 1+\frac{1}{16\epsilon}\left(1-c_a\right)^2 > 0,$$

and

$$\frac{\partial\mathbb{E}\left[v\left(x,k,p\right)\right]}{\partial k} = -\frac{1}{16\epsilon}\left(1-c_a\right)^2 < 0.$$

On the other hand, when consumers are at optimum, but the firm is at the corner solution, we have

$$\frac{\partial\mathbb{E}\left[v\left(x,k,p\right)\right]}{\partial x} = 1+\frac{1}{16\epsilon^3}\left[\epsilon^2+3\left(\epsilon-k+x\right)^2\right] > 0.$$

It is equally straightforward to compute that

$$\frac{\partial\mathbb{E}\left[v\left(x,k,p\right)\right]}{\partial k} = -\frac{3}{8\epsilon^2}\int_{x-\epsilon}^{x+\epsilon}\mu\left(k\right)^2 d\theta < 0.$$

Combining the results of both cases, we get that $\frac{\partial\mathbb{E}[v(x,k,p)]}{\partial x} > 0$ and $\frac{\partial\mathbb{E}[v(x,k,p)]}{\partial k} < 0$ hold everywhere. In words, expected payoff is increasing in own type and in the number of other people playing $a=B$ (i.e. decreasing in the cut-off signal used by other people).

Let $\kappa(k)$ be a point at which $\mathbb{E}[v(x,k,p)] = \mathbb{E}[v(\kappa(k),k,p)] = 0$. This means that the best response to a switching strategy with a cut-off point k is a switching strategy with a cut-off point $\kappa(k)$. In equilibrium, we must have $\kappa(k) = k$. By induction, strategy $s(x)$ survives n rounds of iterated deletion of strictly dominated strategies if

$$s(x) = \begin{cases} N, & x < \underline{\xi}_n \\ B, & x > \overline{\xi}_n, \end{cases}$$

where $\underline{\xi}_0 = -M$ and $\overline{\xi}_0 = M$, and where $\underline{\xi}_n$ and $\overline{\xi}_n$ are defined inductively by

$$\underline{\xi}_{n+1} = \min\left\{x : \mathbb{E}\left[v\left(x, \underline{\xi}_n, p\right)\right] = 0\right\} \tag{28}$$

and

$$\overline{\xi}_{n+1} = \max\left\{x : \mathbb{E}\left[v\left(x, \overline{\xi}_n, p\right)\right] = 0\right\}. \tag{29}$$

First, let us assume that this holds for n rounds. If $a = B$ is the best response to a strategy that has survived n rounds of iterated deletion of strictly dominated strategies, then $a = B$ must be a best response to a strategy with a cut-off rule $\underline{\xi}_n$. The minimal signal x where this holds is defined as $\underline{\xi}_{n+1}$, i.e. $\mathbb{E}\left[v\left(\underline{\xi}_{n+1}, \underline{\xi}_n, p\right)\right] = 0$ holds as proposed in (28). Similarly, if strategy $a = N$ is the best response to a strategy that has survived n rounds of iterated deletion of strictly dominated strategies, then it must be the best response to a strategy with a cut-off $\overline{\xi}_n$. Cut-off point $\overline{\xi}_{n+1}$ is defined as the maximal signal for which this holds.

Since $\mathbb{E}[v(x,k,p)]$ is continuous and strictly increasing in x and strictly decreasing in k, the sequences $\underline{\xi}_n$ and $\overline{\xi}_n$ are monotone. The sequence $\underline{\xi}_n$ is increasing, with $\underline{\xi}_0 = -M < \underline{\theta} < \underline{\xi}_1$, where $\underline{\theta}$ is the boundary value for the lower dominance region defined in Condition 3(i). Similarly, $\overline{\xi}_n$ is a decreasing sequence, with $\overline{\xi}_0 = M > \overline{\theta} > \overline{\xi}_1$, where again, the boundary value $\overline{\theta}$ is defined as in Condition 3(ii). As the number of iterations grows $n \to \infty$, the sequences converge $\underline{\xi}_n \to \underline{\xi}$ and $\overline{\xi}_n \to \overline{\xi}$ due to increasing differences and strategic complementaries $\left(\frac{\partial \mathbb{E}[v(x,k,p)]}{\partial x} > 0 \text{ and } \frac{\partial \mathbb{E}[v(x,k,p)]}{\partial k} < 0\right)$, continuity of $\mathbb{E}[v(x,k,p)]$ and the construction of $\underline{\xi}$ and $\overline{\xi}$. Thus, we get $\mathbb{E}\left[v\left(\underline{\xi}, \underline{\xi}, p\right)\right] = 0$ and $\mathbb{E}\left[v\left(\overline{\xi}, \overline{\xi}, p\right)\right] = 0$.

Next we establish that $\underline{\xi}$ and $\overline{\xi}$ coincide. When the equality is true, there is a unique switching strategy with a unique switching point. The probability that a consumer observes a signal higher than k, when the true state is θ, was given by $\mu(k) = 1 - \frac{k-\theta+\epsilon}{2\epsilon}$. By the law of large numbers, the fraction of other people who observe signals higher that k is less than q when

$$\begin{aligned} & q \geq 1 - \tfrac{k-\theta+\epsilon}{2\epsilon} \\ & \Rightarrow \theta \leq k + 2\epsilon q - \epsilon. \end{aligned} \tag{30}$$

Write the probability (the consumer assigns to the event) that a proportion less than q of other people observe signals higher than k, when the consumer has observed signal x

$$\Psi(q,x,k) = \begin{cases} 1, \quad k + 2\epsilon q - \epsilon > x + \epsilon \\ \int_{\theta=x-\epsilon}^{k+2\epsilon q-\epsilon} h(\theta \mid x)\, d\theta, \quad x - \epsilon \leq k + 2\epsilon q - \epsilon \leq x + \epsilon \\ 0, \quad k + 2\epsilon q - \epsilon < x - \epsilon \end{cases} \tag{31}$$

where $h(\theta \mid x) = \frac{1}{2\epsilon}$ is the density the consumer assigns to state θ when he has observed signal x. The feasible integration range of θ in equation (31) is obtained from system (30). When x is observed, states farther than ϵ away from x are assigned zero density. Integration gives

$$\Psi(q,x,k) = \frac{1}{2\epsilon}(k + 2\epsilon q - x),$$

for $x - \epsilon \leq k + 2\epsilon q - \epsilon \leq x + \epsilon$. In equilibrium $x = k$ as the iterated deletion of strictly dominated strategies suggests. The probability becomes an identity function $\Psi(q,x,x) = q$. The probability $\Psi(q,x,x)$ is also the cumulative distribution function of q on the unit interval $[0,1]$. It is now seen that the distribution of q is uniform on support $[0,1]$, with density $\psi(q) = 1$. The expected utility for action $a = B$ versus $a = N$, when the expected fraction q of neighbours choose $a = B$, is therefore

$$\begin{aligned} \mathbb{E}[v(x,q,p)] &= \int_{q=0}^{1} \psi(q)\, v(x,q,p)\, dq \\ &= \int_{q=0}^{1} v(x,q,p)\, dq. \end{aligned}$$

The indifferent type (consumer with signal x) in equilibrium is given by $\mathbb{E}[v(x,x,p)] = 0$. Hence, by the fact that there is a unique solution $\widetilde{x}$ to $\int_{q=0}^{1} v(\widetilde{x},q,p)\, dq = 0$, the expressions for expected payoff can be equated. As a result, the equilibrium strategy has a unique switching point $x = \widetilde{x}$, which is given by equation

$$\int_{q=0}^{1} v\left(\widetilde{x}, q, p\right) dq = 0.$$

The surviving equilibrium switching strategy is

$$s^{*}\left(\widetilde{x}\right) = \begin{cases} a = B, \text{ if } x > \widetilde{x} \\ a = N, \text{ if } x < \widetilde{x} \end{cases}.$$

■

Proof of Proposition 5. Write the expected profits (17) as

$$\begin{aligned}\mathbb{E}\left(\Pi\right) = \frac{1}{2M} & \left[\int_{\widetilde{x}-\epsilon}^{\widetilde{x}+\epsilon} q\left(\theta, p\right)\left(p - c_f\right) d\theta + \int_{\widetilde{x}+\epsilon}^{M} p - c_f d\theta + \right. \\ & + \int_{\widetilde{x}-\epsilon}^{\widehat{\theta}} q\left(\theta, p\right)^2 \left[1 - c_a - q\left(\theta, p\right)\right] d\theta + \int_{\widehat{\theta}}^{\widetilde{x}+\epsilon} \tfrac{1}{4}\left(1 - c_a\right)^2 q\left(\theta, p\right) d\theta + \\ & \left. + \int_{\widetilde{x}+\epsilon}^{M} \tfrac{1}{4}\left(1 - c_a\right)^2 d\theta\right].\end{aligned}$$

By differentiating the above expression with respect to p, we get the FOC

$$\frac{\partial \mathbb{E}\left(\Pi\right)}{\partial p} = -2p + M + c_f - \tau\left(c_a\right) - \frac{1}{4}\left(1 - c_a\right)^2 = 0. \tag{32}$$

The optimal price is

$$p^{*} = \frac{1}{2}\left(M + c_f\right) - \frac{1}{2}\tau\left(c_a\right) - \frac{1}{8}\left(1 - c_a\right)^2.$$

It is seen directly from the FOC (32) that second order condition for local maximum is satisfied

$$\frac{\partial^2 \mathbb{E}\left(\Pi\right)}{\partial p^2} = -2 < 0.$$

Because the first period profits maximisation problem is unconstrained, p^{*} gives the global maximum. ■

Proof of Proposition 7. We write the profit function with optimal price structure as

$$\begin{aligned}\mathbb{E}\left[\Pi\left(p^*,t^*\right)\right] = & \frac{1}{2M}\left[\int_{\widetilde{x}(p^*)-\epsilon}^{\widetilde{x}(p^*)+\epsilon} q\left(\theta,p^*\right)\left(p^*-c_f\right)d\theta+\int_{\widetilde{x}(p^*)+\epsilon}^{M} p^*-c_f d\theta+\right. \\ & +\int_{\widetilde{x}(p^*)-\epsilon}^{\widehat{\theta}(p^*)} q\left(\theta,p^*\right)^2\left[1-c_a-q\left(\theta,p^*\right)\right]d\theta+ \\ & +\int_{\widehat{\theta}(p^*)}^{\widetilde{x}(p^*)+\epsilon}\frac{1}{4}\left(1-c_a\right)^2 q\left(\theta,p^*\right)d\theta+ \\ & \left.+\int_{\widetilde{x}(p^*)+\epsilon}^{M}\frac{1}{4}\left(1-c_a\right)^2 d\theta\right].\end{aligned} \tag{33}$$

To see the effect of an increase in the precision of signals, we differentiate (33) with respect to ϵ. By applying the envelope theorem, we get the reported result

$$\frac{\partial\mathbb{E}\left[\Pi\left(p^*,t^*\right)\right]}{\partial\epsilon}=-\frac{\left(1-c_a\right)^4}{192M}<0.$$

■

8.3 Vertical separation: perfect competition in second period

The introduction of competition in the second period does not change the solution process. The coordination game satisfies global game conditions.

In the second period price is $t=c_a$, which gives indirect usage utility

$$\lambda^*\left(q\right)=\begin{cases}\frac{1}{2}\left(1-c_a\right)^2 q,\ q\geq 1-c_a \\ q\left(q-\frac{1}{2}q^2-c_a q\right),\ q<1-c_a\end{cases}.$$

The marginal signal is

$$\widetilde{x}=p+\tau_C\left(c_a\right),$$

where $\tau_C\left(c_a\right)=\frac{1}{24}\left(1-c_a\right)^2\left[\left(1-c_a\right)^2-6\right]\leq 0$. In calculating the marginal signal, we need to take into account the cut-off point $q=1-c_a$.

The firm does not take into account whether consumers are constrained in the second period or not. The firm maximises expected first period profits

$$\mathbb{E}\left(\Pi_1\left(p\right)\right)=\frac{p-c_f}{M}\left(M-p-\tau_C\left(c_a\right)\right).$$

The monopoly price equals

$$p_C^* = \frac{1}{2}\left(M + c_f - \tau_C\left(c_a\right)\right).$$

The second order conditions are satisfied, $\frac{\partial^2 \mathbb{E}(\Pi_1(p))}{\partial p^2} = -\frac{1}{M} < 0$. If we plug the optimal price back to the demand function, we get

$$q\left(p_C^*\right) = \frac{\theta^* + \epsilon - \frac{1}{2}M - \frac{1}{2}c_f - \frac{1}{2}\tau_C\left(c_a\right)}{2\epsilon},$$

where θ^* is the realisation of the state θ. If we compare $q\left(p_C^*\right)$ with the monopoly demand of the main model

$$q\left(p^*\right) = \frac{\theta^* + \epsilon - \frac{1}{2}M - \frac{1}{2}c_f - \frac{1}{2}\tau\left(c_a\right) + \frac{1}{8}\left(1 - c_a\right)^2}{2\epsilon},$$

we see that demand is higher with competition in the second period, $q\left(p_C^*\right) > q\left(p^*\right)$. This happens because the monopoly restricts demand in the second period, which reduces expected usage utility.

9 Supplementary section: social relations approach

In the main analysis we adopted a "global" way of looking at network externalities. Each consumer has a need to interact with *any randomly chosen* person from the rest of the population. In other words, each consumer is linked with everyone else. In the terminology of graph theory, the population is characterised by a complete graph of social relations. As a result, we can model network externalities with a function that captures the relevant properties of interaction. A complete graph, however, generates the maximal value for a given number of network members, and therefore we risk overestimating network effects when the true network is something less connected (Sääskilahti 2005).

The class of network models, so called economics of social relations, that has emerged as a refinement to the conventional approach to network effects starts by explicitly considering who is connected to whom. Agents have varying number of connections, depending on the network they belong to. If we think of personal social relations, it is obvious that some people have more connections (large family, a lot of friends) whereas some people are more introvert and maintain only few close relationships. Importantly, no-one knows each member in the society. Because agents lack connections with members of the network, they cannot have a need to interact *actively* with them. Thus, criticism on the functional form approach to network externalities is valid.

There is a rapidly growing literature on social relations. Applications range from job search and unemployment (Calvó-Armengol & Jackson 2004, Bramoullé & Saint-Paul 2004), to wage differentials between employees (Bentolila et al. 2004, Labini 2004), to public good provision (Bramoullé & Kranton 2004), buyer-seller networks (Kranton & Minehart 2001) and R&D cooperation (Goyal et al. 2003), to risk sharing (Goldstein et al. 2002) and social learning (Gaduh 2002) in village economies, all the way to crime (Glaeser et al. 1996, Ballester et al. 2004). Chwe (2000) studies how political action diffuses in social networks when agents use the network to communicate their willingness to adopt a revolutionary action. His work is analogous to product diffusion, where certain consumers (rich or pro new technology) buy the product early and who are followed by mass market adoption. Our related work is a model of monopoly pricing of network goods (Sääskilahti 2005). The focus of that article is on how asymmetric social relations affect monopoly price.

Local interaction models comprise a related field of study (see e.g. Young 1998 ch.6). These models focus on the equilibrium selection in dynamic settings where boundedly rational agents interact with a subset of the total population. Agents take myopic actions in a coordination game with exogenous payoffs (no price setting problem). Agents are only imperfectly rational as an occurrence of a mutant agent (who chooses actions randomly) is positive over time.

The complete graph structure incorporated in the model in the main text usually serves as the benchmark case in the social relations literature. Models of social relations focus on (i) asymmetric networks where some agents have more connections than others, and consequently on (ii) local interaction networks.

Asymmetry between consumers induces the firm to treat consumers differently. Central people with more connections are more important to the whole network, and tend to capture higher surplus than more peripheral consumers (Sääskilahti 2005). In some networks, asymmetry in the number of connections regularises as the network size grows (random graphs), whereas in other networks it magnifies (scale-free networks) (see Albert & Barabási 2002 for technical review, Barabási & Bonabeau 2003 for informal discussion, and Sääskilahti 2005 for an application to a monopoly pricing problem). Regularisation means that the network, despite being asymmetric, presents a priori regular characteristics. In particular, the number of links each node has in a random graph follows a Poisson distribution. Thus, the average number of links is well-defined. Scale-free networks lack such regular statistical properties.

If we confine our analysis on symmetric local interaction models, it turns out that the model in the main text coincides with a social relations approach. That is proved here.

There is a mass I of consumers who are exogenously arranged on a graph $\mathcal{G}$ so that each consumer is located on a unique node of $\mathcal{G}$. We normalise $I = 1$ and treat it as continuum as in the main model. The set of undirected edges (links) on $\mathcal{G}$ is $\mathcal{E}$. Two consumers i and j are neighbours if they are connected by an edge, $\{i, j\} \in \mathcal{E}$. The edges are undirected so that, if $(i, j) \in \mathcal{E} \Rightarrow (j, i) \in \mathcal{E}$. The set of consumer i's neighbours is $\mathcal{N}_i$, with $\mathcal{N}_i \neq \emptyset$ so that there are no isolated nodes (i.e. the network is completely connected)[7] .

Assumption A1 The graph $\mathcal{G}$ is completely connected, symmetric and n-dimensional.

Assumption A1 means that every consumer has $n \in]0, 1]$ neighbours. Consumer i's neighbourhood $\mathcal{H}_i$ is defined as a collection of i and the set of his neighbours, $\mathcal{H}_i = \{i, \mathcal{N}_i\}$. We can allow each consumer's neighbourhood to consist of all other consumers, $\mathcal{H}_i = \{i, \mathcal{G} \diagdown i\}$. This case corresponds to the global interaction model presented in the main text. Since the graph is infinitely large (because consumers are weightless), link configuration with everybody holding identical number of links is guaranteed to exist.

Opposed to the main model, each consumer is now interested only in interacting with his own

[7] Note the difference between complete graph (= everybody is linked with everyone else) and completely connected network (= there is a path between any two network members expanding one or more links).

neighbours only. A link is said to be potentially active if both end nodes have bought the product. Interaction between two consumers is represented by the activation of the link between them. Activation is a directed process, so that both end nodes can activate the same link, but only if both have got the product. Reception of an activated link is automatic.

The problem for consumer $i \in \mathcal{G}$ is to choose action $a_i \in \{B, N\}$, where $B =$ buy the device and $N =$ do not buy. If he chooses $a_i = B$, then he needs to decide which links he activates in the second period. Define an active link between agents i and j as $e_{ij} = 1$. If only one agent buys or neither buy, the edge cannot be activated: $e_{ij} = 0$. A potentially active link is activated if the consumer pays the activation fee t. Activation yields utility which presents decreasing marginal utility. Because social relations are exogenous, the activation need is independent of the number of neighbours who buy. Let $\alpha_i = \frac{\sum_{j \in \mathcal{N}_i} e_{ij}}{n}$ be the fraction of active links per total number of neighbours. We have $\alpha_i \in [0, 1]$. $\alpha_i = 1$ means that all links are activated. Symmetry and exogeneity of social relations guarantee $\alpha_i = \alpha$ for all $i \in \mathcal{G}$. This formulation makes it possible that the consumer would like to activate more links than there are potentially active links. Reception of an active link is free of charge and does not give utility.

Marginal usage utility is $\frac{\partial \lambda(\alpha,t)}{\partial \alpha} = q(1 - \alpha - t)$, where $q \in [0, 1]$ is the probability that neighbour indexed α has bought the product in the first period and t is the per link activation fee. By law of large numbers, q is the proportion of consumers who bought the device in the first period. Consumer's second period objective is to maximise expected usage utility,

$$\max_{\alpha} \{\lambda(\alpha, t)\}, \text{ s.t. } 0 \leq \alpha \leq q.$$

The optimal level of usage is

$$\alpha^*(t, q) = \min\{1 - t, q\}.$$

Because all consumers are symmetric, firm's second period problem is identical to the one in the main model. In the second period, first period profits and the proportion of consumers who bought the product are fixed, so, we can write second period profits as

$$\Pi_2 = q\alpha^* (t, q) (t - c_a),$$

where $c_a \in [0, 1]$ is the per link activation cost.

The firm charges an activation price such that the consumers are maintained at an efficient level of link activation (see the main text). We can write the firm's second period problem as

$$\max_t \{q\alpha^* (t, q) (t - c_a)\}, \text{ s.t. } t \in [1 - q, 1[.$$

The optimal usage fee is

$$t^* = \max\left\{\frac{1}{2}(1 + c_a), 1 - q\right\}.$$

We have arrived to identical optimal levels α^* and t^* as in the main model. These give us value functions $\lambda^* (t^*, q)$ and $\Pi_2^* (t^*, q)$ which match equations (4) and (3) respectively.

For perfect information regime, we have to assume that all neighbourhoods are identical in order to have all the arguments of the main text go through. Identical neighbourhoods means that in each neighbourhood, consumer types are distributed uniformly over $[\theta - \epsilon, \theta + \epsilon]$. This constraint is quite strong, and it reduces the applicability of the model under perfect information. On the contrary, in the incomplete information regime, we do not need to make any additional assumptions to the main model. If Condition 3 (dominance regions) holds, all the arguments of the main model go through.